An Affordable Approach for Robust Design of Thick Laminated Composite Structure

Wei Chen*
Assistant Professor
Mechanical Engineering
University of Illinois at Chicago
842 W. Taylor, M/C 251
Chicago, IL. 60607-7022
Phone: (312) 996-6072, Fax: (312) 413-0447
email: weichen1@uic.edu

Wei Fu
Staff Engineer
Global Software Division
Motorola Inc.
Former Ph.D. Student at Clemson University

S.B. Biggers
Professor
Mechanical Engineering
Clemson University

R.A. Latour
Associate Professor
Bioengineering
Clemson University

* Corresponding Author

Abstract

A systematic and affordable approach is proposed for the robust design of thick laminated composite structures. Our approach integrates the principles of the Robust Concept Exploration Method (RCEM) for designing complex engineering systems and the hierarchical multi-level optimization procedure for managing the complexity of composite structure optimization. Foundational to the proposed approach is the use of Design of Experiments (DOE) techniques and the Response Surface Methodology (RSM) for improving computational efficiency in using high fidelity design simulations; and the use of the robust design method for improving the quality of a product that is insensitive to potential variations of design parameters. Our approach is illustrated through the design of a laminated composite femoral component for hip joint arthroplasty. The solution yields the robust design of a composite hip implant, which is applicable for a range of bone stiffness, thereby eliminating the need to design specifically for an individual.

Key words: robust design, multi-level optimization, design of experiments, composite structure design, hip implant

1. Introduction

With a thick laminated composite structure, a designer has the freedom to vary the orientation of each ply to achieve beneficial stiffness, stress distributions, subsequent component strength, and physiological performance. However, due to the increased design freedom and the heterogeneous anisotropic nature of structures, composite structures are much more complex to design than their homogeneous isotropic metallic counterparts. Although optimization techniques have been applied to the design of composite structures (Fukunaga and Vanderplaats, 1991; Conti and Cella, 1992), the majority of the published work in this area concentrates on the optimum design of *thin* laminates for specific individual components, such as plates, shells and beams. Difficulties associated with the design of *thick* laminated composite structures arise from the following aspects:

- *Large number of design variables*: A thick laminated composite structure is made up of many thin layers (plies) orientated at various angles. Thus, even a relatively simple composite plate with each ply orientation as a design variable has many more design variables than a comparable isotropic plate. This complicates the optimization process and increases the difficulty in providing an overall view of the design space.
- *Highly nonlinear nature*: The high degree of interdependence of the design variables, and the highly nonlinear nature of the expressions relating structural performance attributes and design variables, may cause convergence difficulties for large-scale problems (Watkins and Morris, 1987).
- *Huge computational demand*: The behavior of thick laminated composite structures can hardly be expressed using analytical forms. Significant computational resources

are required to conduct high fidelity finite element analysis using programs such as ABAQUS®. Even with state-of-the-art computer systems, such analysis can easily take close to an hour or more for a single simulation. Optimization using high fidelity analysis is hence difficult to manage.

In practice, decomposition and multi-level optimization schemes are often used to manage the complexity of structural optimization problems. Sobieski and James (1985) developed a method for decomposing an optimization problem into a set of subproblems to reduce the design time on a large design problem. The decomposition is achieved by separating the structural element optimization subproblems from the assembled structural optimization problem. Watkins and Morris (1987) used a multilevel optimization scheme and also a multicriteria objective function optimization for large laminated composite structures to minimize the weight function and strain energy of the structure. Conti and Cella (1992) described a two level approach in their paper to optimize composite structures which is very useful in conjunction with the finite element technique. In their method, first an overall optimization of an equivalent orthotropic laminate is performed, and then an optimum lay-up sequence search is conducted for the structure under in-plane loading. While the vast portion of the existing methods are only useful for the design optimization of thin 2-D laminates, the authors (Fu 1998; Fu et al. 1998; Srinivasan et al.1996) developed a hierarchical three-level optimization scheme for fully 3-D thick composite structure to desired levels of structural stiffness and maximum strength.

The existing multi-level optimization schemes help to reduce the total number of design variables and the nonlinearity of the performance attributes, however, the optimization approach for thick laminated composite structures design is still not widely used because of the huge computational demand for high fidelity analysis. Methods are therefore needed to further improve the applicability of optimization approach in this domain and provide better knowledge of the design space. Furthermore, the approach should assist the development of composite structures whose behavior is robust with respect to the conditions under which they will be used. In bioengineering applications, an example of these conditions could be associated with the properties of human bones.

Our aim in this paper is to develop a systematic and *affordable* approach for thick laminated composite structure optimization that integrates the principles of the Robust Concept Exploration Method (RCEM) for designing complex engineering systems (Chen et al. 1997) and the hierarchical multi-level optimization procedure for managing the complexity of thick composite structure optimization (Fu et al. 1998). The RCEM was developed for quickly evaluating the design alternatives and determining design specifications with quality considerations for designs involving high fidelity analysis. The RCEM approach has been tested for various engineering design problems (Chen et al. 1997; Simpson et al. 1996; Koch et al. 1996; Peplinski et al. 1996; Lautenschlager et al. 1996; Bailey et al., 1997). These preliminary studies of the RCEM illustrated that the RCEM can be used to greatly enhance the optimization capability and facilitate fast convergence of the solution for designs involving high fidelity analysis. Using the quality engineering concept, it permits the introduction of downstream design considerations in the early stages of design and provides flexibility in the design process.

In this work, the principles of the RCEM are incorporated into a hierarchical multi-level optimization procedure for thick laminated composite structure design that facilitates solutions to multi-objective problems by decomposing the problem into sub-problems.

Our approach is demonstrated for the robust design of a laminated composite femoral component for hip joint arthroplastry. The robust design procedure will generate design specifications that are applicable for a range of bone factors, thereby eliminating the need to design components specifically for an individual.

2. Designing Thick Laminated Composite Structure Based on RCEM Principles

The fundamental principles of the RCEM include the use of Design of Experiments (DOE) techniques (Box et al., 1978; Montgomery, 1991) and the Response Surface Methodology (RSM) (Box and Draper, 1987; Khuri and Cornell, 1987) for improving computational efficiency in designing complex systems; and the use of the robust design method for improving the quality of a product that is insensitive to potential variations of design parameters.

DOE techniques are used to conduct a set of numerical experiments in which the input variables are systematically changed to observe the behavior of the output responses. Associated with the DOE, response surface methodology (RSM) generates mathematical functions for constructing global approximations to system behavior. DOE and RSM can be used to construct global and midrange approximations (surrogate models) to functions in structural optimization to achieve rapid convergence. The strength of the method is in applications where the design simulation is computationally expensive, the calculation of the design sensitivity information is difficult or impossible to compute, as well as in cases with “noisy” functions, where the sensitivity information is not reliable, or when the function values are inaccurate (Venter et al.; Unal et al, 1996).

The robust design principle was originally developed by Taguchi to improve the quality of a product through minimizing the effect of the causes of variation by changing the settings of controllable design variables (Taguchi, 1994; Phadke, 1989). Following this principle, a good design is defined as one that not only achieves its best performance but also minimizes the performance variance. The adaptation of Taguchi's robust design principle in RCEM relies on a general robust design procedure developed by (Chen et al., 1996) for solving two broad categories of robust design problems. These are, Type I–robust design associated with the minimization of variations in performance caused by variations in noise (uncontrollable) factors and Type II–robust design associated with the minimization of variations in performance caused by variations in control factors (design variables). Type I robust design is particularly relevant to this work.

The principles of the RCEM are employed in this work to develop a systematic and affordable approach for thick laminated composite structure design. Figure 1 is a flow chart of the proposed procedure. The major components of this infrastructure include four processors (modules B, D, E and F) and a simulator (module C). The central slot, module C, is the ABAQUS based high fidelity finite element simulation program. The different processors correspond to the four major steps of the proposed procedure. They are:

Step 1: Classify system parameters (Module A)

Step 2: Build response surface models (Modules B, C)

Step 3: Identify significant factors (Module D)

Step 4: Apply robust design techniques (Module F)

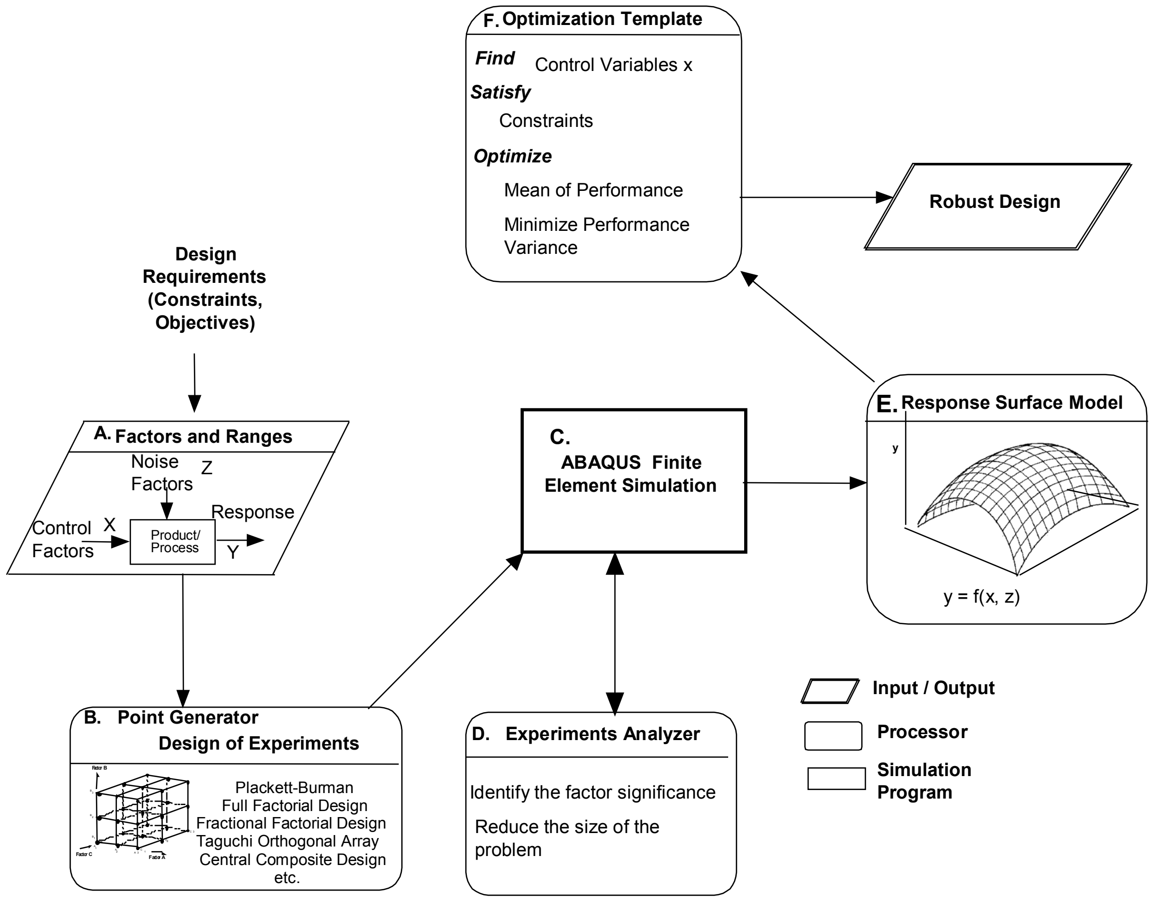


Figure 1. Computer Infrastructure of the Proposed Approach

In Step 1, based on the principles of quality engineering, design parameters of the laminated composite structure are classified as the control factors (design variables), the noise factors (uncontrollable variables), and the responses (performance). Step 2 is to build the response surface models used to replace the expensive analysis programs. Module B and the simulator C perform computer experiments in a systematic manner. The results are analyzed in module D and the response surface model is created in module E. In general, the response surface model is represented by the following equation:

$$y = f(x, z) \tag{1}$$

where y is the estimated response, x represents the design variables, and z represents the noise factors. Depending on the desired order of the response surface model, different types of experiments are chosen by module B to achieve the best accuracy of the surrogate models.

Step 3 involves identifying the significant factors of the response surface models based on the results of the statistical analysis. The percentage contribution of each factor is determined to assess their importance. The factors with a low percent contribution are deemed trivial and could be eliminated from the optimization procedure, thereby reducing the size of the problem.

Based on the reduced response surface models, in Step 4, the robust design method is applied to generate design solutions that are robust to potential design deviations (module F). For a typical structure optimization model that is stated in Eqn. (2):

$$\begin{aligned} &\text{Find} && x \\ &\text{minimize} && f(x) \\ &\text{subject to} && g_j(x) \le 0, \qquad j = 1,2,....,J \\ &&& x_L \le x \le x_U, \end{aligned} \tag{2}$$

The robust design can be formulated as a multiobjective optimization problem shown as the following:

$$\begin{aligned} &\text{Given:} && y = f(x, z), \text{ deviation of noise parameters } \mu_z, \sigma_z \\ &\text{Find:} && x \\ &\text{Minimize:} && [\mu_f, \sigma_f] \\ &\text{s.t.} && g_j(x) + k_j \sum_{i=1}^{n} \left| \frac{\partial g_j}{\partial z_i} \right| \Delta z_i \le 0, \qquad j = 1,2,....,J \\ &&& x_L \le x \le x_U, \end{aligned} \tag{3}$$

where μ_f and σ_f are the mean and the standard deviation of the objective function $f(x)$, respectively. The problem is formulated as a multiobjective optimization problem in which the goal is to simultaneously optimize the mean of the performance f and minimize its variations, subject to the feasibility of constraints $g_i(x)$ under deviations. To ensure the feasibility of the constraints under the deviations of the design variables, we use the worst case scenario, which assumes that all variations of system performance may occur simultaneously in the worst possible combination of design variables (Parkinson et al, 1993). The original constraints are modified by adding the penalty term to each of them, where k_j is a constant, chosen by the designer, that reflects the compensation of the error in estimating the worst case when using the first-order Taylor expansion. Depending on the computation resource, μ_f and σ_f could be obtained through simulations or analytical means such as Taylor expansions. When using Taylor expansions, these functions can be represented by the following equations:

Mean of the response: $$\mu_{\hat{y}} = f(x, \mu_z) \tag{4}$$

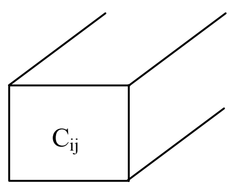
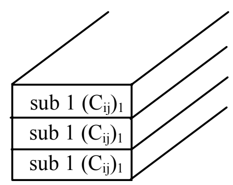
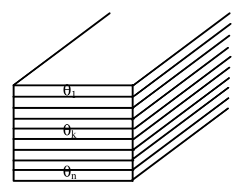
Variance of the response: $$\sigma_{y}^{2} = \sum_{i=1}^{k} \left(\frac{\partial f}{\partial z_i} \right)^2 \sigma_{z_i}{}^{2} \tag{5}$$

Multiobjective optimization formulations could then be employed to capture the tradeoff between optimizing the mean performance μ_f and minimizing the performance variance σ_f.

3. The Multi-Level Design Scheme

It has been discussed in Section 1 that the large number of design variables present in the thick composite structures complicates the optimization design procedure and increases the difficulty of convergence. Even with the approach of using response surface models as surrogate models, to achieve a good model accuracy, it is desirable to keep the number of design variables at its minimum. For this purpose, a multi-level optimization scheme developed by the authors (Fu, 1998; Fu, et al., 1998) is adopted as the design framework in our proposed approach. As shown in Table 1, three optimization levels are considered for a 3-D thick composite structure. In the first level (laminate level), the orthotropic laminate stiffness coefficients for the overall structure, C_{ij}, are taken as design variables to optimize the laminate in-plane stiffness under in-plane loading. In the second level (sublaminate level), the laminate is divided into sublaminates and the stiffness coefficients of the orthotropic sublaminates, $(C_{ij})_k$, are taken as design variables to optimize the laminate out-of-plane stiffness under out-of-plane loading while keeping the overall in-plane performance as optimized in Level 1. For these first two levels, a global finite element model of the composite structure and the femur (the long thigh bone) is developed with effective (or smeared) laminate/sublaminate stiffnesses to obtain the desired global structural behavior. In the third level, a local finite element model in a designated region of interest is used to accurately compute the ply level stresses. In this optimization level (ply level), optimum stacking and orientation of each ply are determined to maximize the laminate strength according to both intralaminar and interlaminar theories while maintaining the overall sublaminate stiffness coefficient values as determined in Level 2. This procedure thus enables the laminated composite structure to be globally designed to provide the desired in-plane and out-of-plane stiffness and the resulting in-plane and out-of-plane response, while also locally designing for maximum static strength.

Table 1. Summary of Models at Different Levels (Fu et al., 1998)

Optimization Level:	Level 1 (laminate)	Level 2 (sublaminate)	Level 3 (ply)
Structural Model:	Global model, homogeneous stem and surrounding bone, in-plane loading, 3-D	Global model, piece-wise homogeneous stem and surrounding bone, in-plane & out-of-plane loading, 3-D	Local model, individual plies, 3-D
Objective:	Strain energy in the calcar region of the femur	Interfacial shear stress between of the femur and the stem	Stem strength
Design Variables:			
Constraints:	Interfacial shear stress between the bond and the stem	In-plane stiffness	In-plane stiffness and out-of-plane stiffness

For illustration purposes, the proposed approach is only applied for the first and second level optimizations in this paper. At both levels, the principles of the RCEM are employed to create the surrogate models for replacing the high fidelity finite element analysis. Subsequently, the composite structures whose behaviors are robust with respect to the conditions under which they will be used are developed based on the robust design techniques. The proposed approach can be easily extended to Level 3 without bringing up new computational issues. Detailed procedures and results for the three level optimization scheme proposed in Table 1 can be found in reference (Fu, 1998), where the computational issue in Level 3 has been taken care of by using a local finite element model.

4. Laminated Composite Femoral Component for Hip Joint Arthroplasty

4.1 Problem Description

The proposed procedure is illustrated by application to the design of a simplified representation of a laminated composite femoral component for hip joint arthroplasty. The use of low-stiffness composite materials is investigated to overcome bone remodeling problems associated with stress shielding induced by the high stiffness of the metallic implants (titanium or cobalt chromium alloys) (Kumar and Tauchert, 1992). In a laminated composite stem, the designer has the freedom to vary the orientation of each ply to achieve beneficial stiffness, stress contributions, subsequent component strength, and physiological performance. However, due to this increased design freedom and their heterogeneous anisotropic nature, composite stems are much more complex to design than their homogeneous isotropic metallic counterparts. The design problem for the composite component would be to design an optimally stacked composite component to achieve optimal in-plane and out-of-plane stiffness so that the stress shielding of the surrounding bone can be minimized, the interfacial shear stress between the implant and the surrounding bone can be minimized, and the stem strength can be maximized at the same time. The schematic of a simplified structural model is provided in Figure 2. The structural model is the global model representing a simplified human femur with its lower end fixed and the upper region containing a modeled composite femoral component inserted within the femur's central canal and extending out of the top. The top of the component extends above the femur to represent the femoral neck and head of the device.

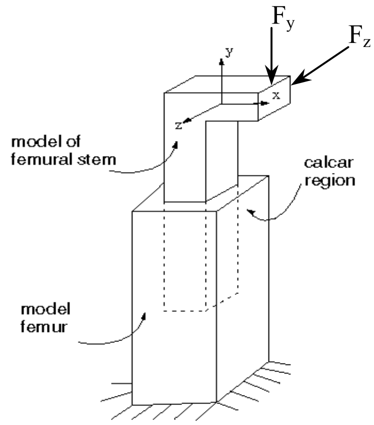


Figure 2. Schematic of Structural Model

In this simplified model, the composite component consists of 60 plies. Each ply is a layer made of carbon fiber reinforced polyetheretherketone (CF/PEEK) with the material properties listed in Table 2. The bottom of the model is fixed, the in-plane (x-y plane) loading applied is 1000N in the negative y-direction, and the out-of-plane loading applied is 100N in the z-direction. Based on the multi-level optimization scheme discussed in Section 3, the optimization models at levels 1 and 2 are particularized and their relationship is presented using the following diagram (Figure 3).

At Level 1 (laminate level), the laminated composite structure is optimized in terms of its global stiffness coefficients (C11, C22, C13, C44, C55). The logistics behind the choice of these five coefficients among the nine composite design coefficients are discussed in (Fu et al., 1998) and will not be repeated here. The structure is treated as a homogeneous (balanced and symmetric) orthotropic laminate under in-plane loading. The strain energy

E in the upper-most central portion of the femoral calcar region (Figure 1) is optimized while maintaining the stem/bone interfacial shear stress for in-plane loading SS1 within its limit (8.5 psi). The calcar region is selected since it is the area most subject to stress shielding induced bone resorption following hip arthroplasty. The strain energy is the function identified to quantify the stress shielding in the calcar region.

Table 2. Material Properties of CF/PEEK Composite [APC-2/AS4]

(ICI Thermoplastic Composite Inc., 1991)

In-plane longitudinal modulus	E_{11} = 135.3 GPa
In-plane transverse modulus	E_{22} = 9.0 GPa
Out-of-plane modulus	E_{33} = 9.0 GPa
In-plane shear modulus	G_{12} = 5.2 GPa
Out-of-plane shear modulus	G_{13} = 5.2 GPa
Out-of-plane shear modulus	G_{23} = 1.9 GPa
Possion's ratio	$V_{12} = V_{13}$ = 0.34, V_{23} = 0.46
Longitudinal strength	X = 2068 MPa
Transverse strength	Y = 86 MPa
Peel strength	Z = 86 MPa
In-plane shear strength	S = 188 MPa

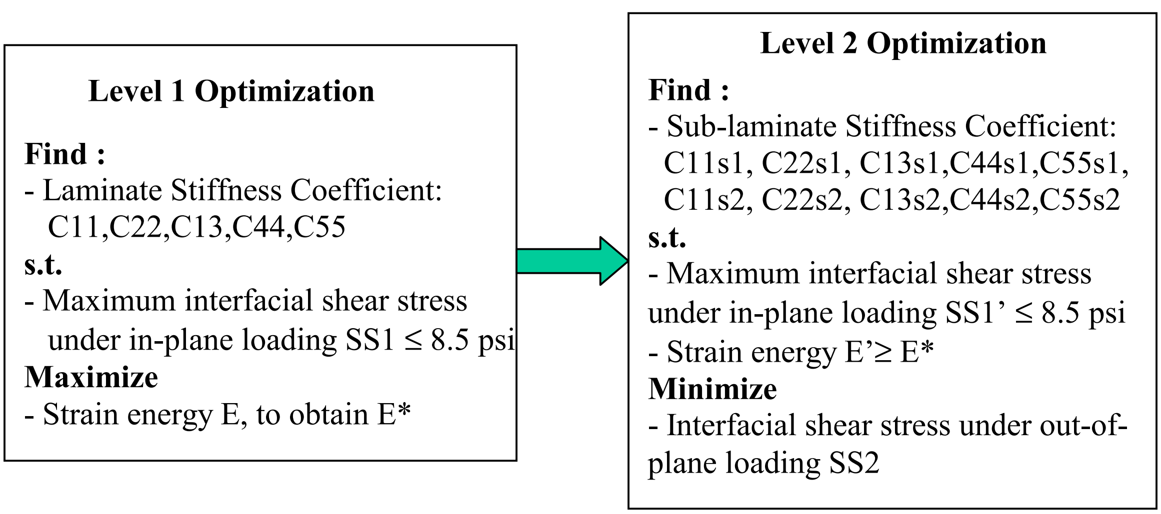


Figure 3 Bi-Level Optimization Scheme

In the second level (sub-laminate level), each composite material is separated into two sub-laminates and the design variables are the sublaminate stiffness coefficients (C11s1, C22s1, C13s1, C44s1, C55s1, C11s2, C22s2, C13s2, C44s2, and C55s2). The interfacial shear stress for out-of-plane loading SS2 is optimized while maintaining the performance of Level 1 by constraining the maximum allowable interfacial shear stress for in-plane loading, i.e., SS1' ≤ 8.5 psi, and the strain energy performance of Level 1, i.e., E' ≥ E*. The strain energy performance limit E* is the maximized energy performance at Level 1. The models in Figure 3 are conventional optimization models without any robust design considerations.

To illustrate the use of robust optimization for laminated composite structure design, the bone factor is considered as a parameter subject to variations. The bone is assumed to be an isotropic material with E (Young's Modulus) varying between the range of 17.5 Gpa to 21.9 Gpa for a group of individuals. In general we could also assume that the bone factor of an individual will change with aging based on the new adaptation to the stress field. The techniques of converting a conventional optimization into a robust design formulation (see Eqn. 3) will be used. This is further explained in Section 4.5.

4.2 Classification of Parameters

Step 1 of the proposed procedure is the classification of all parameters into design variables (control factors), noise factors, and system responses. In Level 1, the laminate stiffness coefficients (C11, C22, C13, C44, and C55) are taken as design variables. The bone factor, B, subject to deviations, is considered as the noise factor. The system responses include the strain energy (E) and the interfacial shear stress for in-plane loading (SS1), the two performance attributes used to evaluate constraint and objective. Similarly, in Level 2, the sublaminate stiffness coefficients

(C11s1, C22s1, C13s1, C44s1, C55s1, C11s2, C22s2, C13s2, C44s2, and C55s2) are taken as control factors. The noise factor again is the bone factor, B. The responses include the strain energy E', maximum interfacial shear stress under in-plane loading SS1', and interfacial shear stress under out-of-plane loading SS2.

To conduct the design of experiments, the boundaries of all the factors, including both the control and noise variables, need to be determined first. In Level 1, the boundaries for the laminate stiffness coefficients are set by varying all the ply angles between 0^o and 90^o and initially allowing the coefficients to independently vary within a continuous limited domain (Fu et al., 1998). These boundaries are defined for physically realistic solutions based on the positive definite requirement for the material stiffness matrix. As shown in Figure 4, a set of boundaries is defined to represent the feasible domain for (C11, C22). Since this region is not rectangular in shape and design of experiments can only be created for factors defined by their lower and upper bounds, two rectangular regions are used to replace this feasible domain. Boundaries of these two regions are listed in Table 3 and entitled as Design Region I and Design region II, respectively. Also included is the range of noise factor, B, the bone factors. The boundary conditions are basically the same in regions I and II for all the factors except C11 and C22. Response surface models will be created separately at Level 1 over these two regions and the better robust design solution will be chosen as the final solution for Level 1 robust design. In Level 2, (C11s1 , C11s2), (C22s1, C22s2), (C13s1, C13s2), (C44s1, C44s2), and (C55s1, C55s2) have the same boundaries as that of C11, C22, C13, C44, and C55 in Level 1, respectively. Note for creating response surface models over irregular domains, advanced design of experiment techniques such as D-optimal designs can be used. However, those computational procedures are usually more complicated compared to standard design of experiments used here.

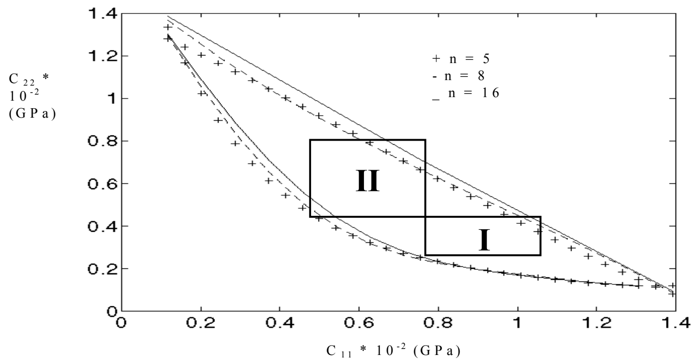


Figure 4. Feasible Domain for C_{11} & C_{22}

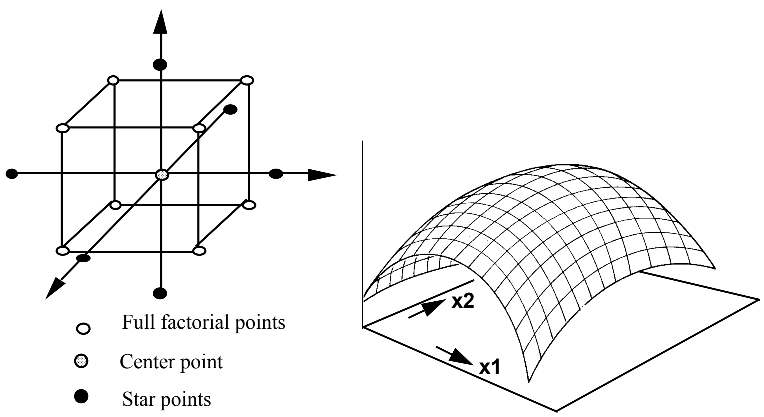
Table 3 Boundary Conditions in Level 1

	Design Region I		Design Region II	
Mpa	low	high	low	high
C11	80000	115000	50000	80000
C22	22000	40000	40000	65000
C13	5495.4	5832.7	5495.4	5832.7
C44	1900	5200	1900	5200
C55	1900	5200	1900	5200
B	17500	21900	17500	21900

4.3 Building Response Surface Models

Second-order response surface models are created at both Levels 1 and 2 as surrogate models representing the relationships between the responses and all the factors, including both design variables and noise factors. In Level 1, standard Central Composite Design (CCD), which generates 77 experiments for six factors, is used to conduct computer simulations. A typical CCD consists of a complete or fraction of a first-order (2^n) factorial design, two star points on the axis of each design variable at a distance α from the center, and one or a number of center points. An example of the Central Composite Design (CCD) for fitting a second order surface model of three variables (factors) is shown in Figure 5. In Level 2, for 11 factors (10 control and one noise), 2,074 experiments are required if using CCD. This is not affordable and the LAT (Latin Hypercube) experiments are used instead. With LAT, random points are picked in the way to have equal appearance along each variable direction. For both regions, 100 LAT experiments are conducted to ensure accurate approximations.

		A	B	C
Factorial portion	1	-1	-1	-1
	2	-1	-1	+1
	3	-1	+1	-1
	4	-1	+1	+1
	5	+1	-1	-1
	6	+1	-1	+1
	7	+1	+1	-1
	8	+1	+1	+1
Star portion	9	$-\alpha$	0	0
	10	$+\alpha$	0	0
	11	0	$-\alpha$	0
	12	0	$+\alpha$	0
	13	0	0	$-\alpha$
	14	0	0	$+\alpha$
Center point	15	0	0	0



$$y = \beta_0 + \sum_i \beta_i x_i + \sum_i \beta_{ii} x_i^2 + \sum_{i \le j} \beta_{ij} x_i x_j$$

Figure 5 CCD and Second-Order Response Surfaces

The accuracy of the response surface models is verified before continuing with robust design optimization. For the points simulated, R-coefficient of regression analysis is the best indication of the model. The closer R is to 1, the better the fit. It is observed that the regression coefficients obtained for all the models at both levels and over both regions I and II range from 0.980 to 0.999, which are all very close to 1. Therefore, the surrogate models are reasonably accurate for all of the responses. Tests have also been conducted to check the accuracy of the models by comparing the results obtained from the response surface model and those using the original analysis program (ABAQUS). An example is provided in Figure 6 for the strain energy E in Level 1 to illustrate the similarity between the actual model and the approximation model using grid plots. It is noted that both grids are within the same range and have similar surfaces.

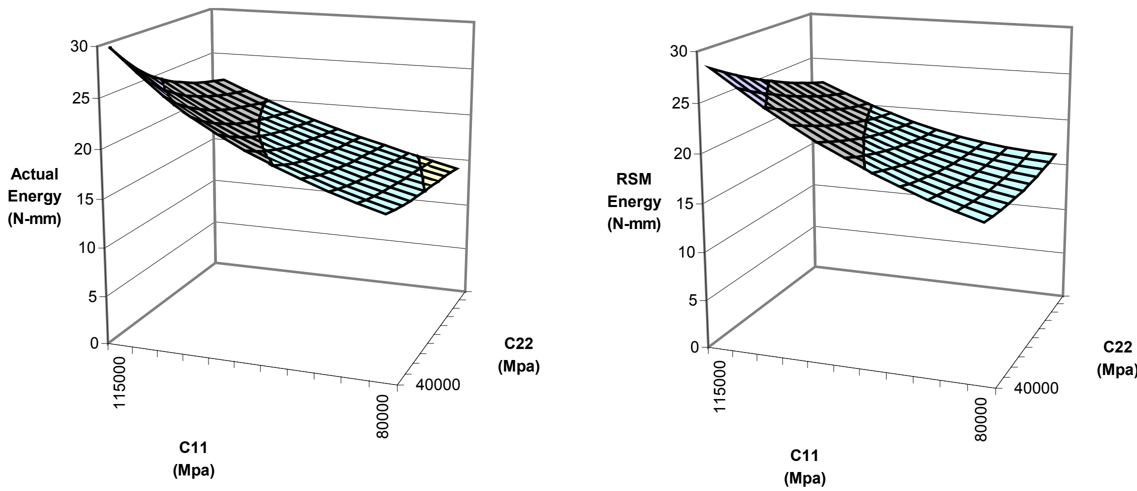


Figure 6. Comparison of the Actual Behavior and the Response Surface Model (Level 1, Design Region I, Strain Energy)

4.4 Identification of Significant Factors

One of the benefits of using second-order response surface models is that after normalization of the design factors, the coefficients of the quadratic equation directly indicate the significance of the first-order effects (linear terms), the interaction effects (interaction terms), and the second-order effects (quadratic terms). This process can help the designer gain insight into the problem, have a better knowledge of the structural behavior, and reduce the size of the problem. In this work, those factors with less than 1% contribution are considered trivial and are omitted from the robust design optimization procedure to reduce the computational complexity. Examples of resulting response surface equations, entitled reduced response surface models, are presented in Appendix. Figure 7 is a bar chart showing the percentage contributions of the design variable and noise factor for Level 1 over both design regions I and II.

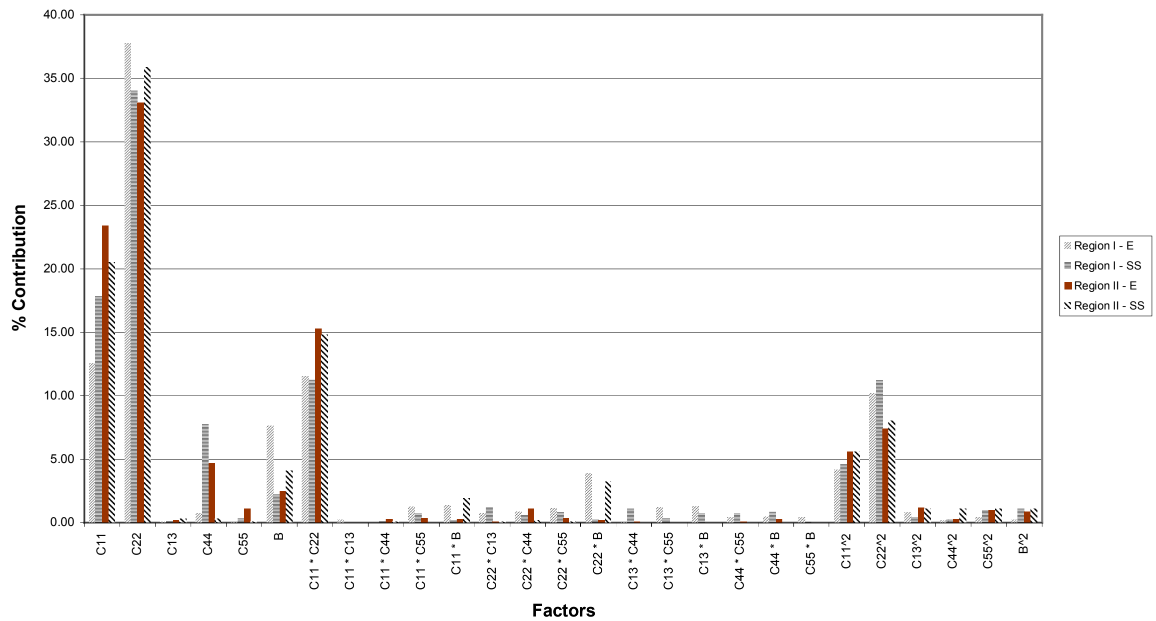


Figure 7. Percentage Contribution of Level 1 Factors

It is observed from Figure 7 that C11 and C22 have the highest contributions to all the responses. These two factors also have stronger interactions with the bone factor, B, compared to other interaction effects. This is reflected in the contributions from C11*B and C22*B. Knowledge of the relationship between control factors and the noise factor is important in robust design as a large interaction indicates the possibility of adjusting the level of a control factor to reduce the impact of a noise factor.

4.5 Robust Design Solution

The reduced response surface models are used to replace the high fidelity analysis programs in robust design. With the multi-level scheme, robust design optimization is conducted sequentially at Levels 1 and 2. The robust design formulation presented in Eqn. (3) is used to convert a conventional optimization problem to its robust design formulation. In Level 1, the variance of objective is derived as:

$$\sigma_E^2 = \left(\frac{\partial E}{\partial B}\right)^2 (\sigma_B)^2 \tag{6}$$

The standard deviation of bone factor (σ_B) is chosen based on the assumption that the bone factor is normally distributed within the given range [17.5-21.9] GPa with $\pm 3\sigma_B$ deviations. Hence σ_B is derived as 0.73 GPa.

To satisfy different preferences as to whether it is more important to optimize the mean performance or to minimize the deviation, different weighting schemes for the normalized objective functions are tested. In Table 4, results based on three different weighting schemes are provided for Level 1. In general observation, Case I, in which the objective of bringing the mean on target is given the highest priority (W1=1, W2=0), generates the best performance of strain energy. This is equivalent to the scenario of optimization without robustness considerations. Conversely, Case 2, in which minimizing the noise variation is given the highest priority (W1=0, W2=1), indicates the best achievable variance (0.25323) and the sacrifice from the mean performance. The results under Case 3 are obtained using an equal weighting function, which represents a tradeoff between the two robust design objectives.

It is observed that the best achievable values for the mean and variance over Region I are 20.30 N-mm and 0.25323, respectively. For Region II, the best achievable values for the mean and variance are 18.09 N-mm and 0.00122, respectively. In comparison with the results without robust design consideration, it is determined that the sacrifice of performance is reasonable for the robust design solution over Region I. However, when variance is introduced in the model within Region II, the mean performance is reduced significantly from 18.09 N-mm to 9.62 N-mm.

Table 4. Robust Design Solutions in Level 1

	Region I	Region II
Case 1 (w1=1; w2=0)		
Energy (N-mm)	20.30	18.09
Shear Stress (psi)	8.41	8.42
Variance	0.40299	0.33558
Stiffness Coefficients (MPa)		
C11	114945	75358.5
C22	27676.9	40020.6
C13	5495.4	5832.7
C44	1900	1900
C55	5200	1900
Case 2 (w1=0; w2=1)		
Energy (N-mm)	17.38	9.41
Shear Stress (psi)	8.32	4.91
Variance	0.25323	0.00122
Stiffness Coefficients (MPa)		
C11	95847.9	78919.5
C22	35847.9	61697.9
C13	5664.11	5664.11
C44	3550	3550
C55	3550	3550
Case 3 (w1=0.5; w2=0.5)		
Energy (N-mm)	19.72	9.623
Shear Stress (psi)	8.45	5.13
Variance	0.27060	0.00122
Stiffness Coefficients (MPa)		
C11	114920	78919.5
C22	28375.9	61697.9
C13	5831.25	5832.7
C44	1900	3929.86
C55	1900	3929.86

Based on these comparisons, the solution over Region I under Case 3 is recommended as the solution to Level 1. It is also noted that except C13, the solutions of other four design variables vary quite a lot under different design scenarios. This indicates that the design solutions are sensitive to the need of minimizing energy variance.

The results obtained from Level 1 model are used to constrain the robust optimization at Level 2. In particular, the achieved energy level E* = 19.72 N-mm is used as the lower bound for the strain energy constraint E' in Level 2. In addition, only the design space over region I is considered to match with the solution from Level 1 (Case 3). The robust design solutions for Level 2 optimization under three cases with different weight settings are summarized in Table 5. It is observed that the best achievable values for the mean and variance of SS2 are 1.38 psi (Case 1) and 0.00013 (Case 2), respectively. Case 3 illustrates a reasonable tradeoff in which the minimized variance (0.00017) is very close to the best achievable variance and the sacrifice of the performance (1.7918) is acceptable. With the robust design consideration, the variance of interfacial shear stress (SS2) under the deviation of the bone factor could almost be considered as 0. As expected, the suboptimal stem exhibits lower Level 1 interfacial stresses and calcar strain energy as a result of its greater in-plane bending stiffness. Considering Level 2, the optimal stem, despite its lower in-plane bending stiffness, is able to be designed with out-of-plane stiffness equivalent to the stiffer suboptimal stem design.

Table 5 Robust Design Solution at Level 2

		Case 1 w(1,0)	Case 2 w(0,1)	Case 3 w(0.5, 0.5)
E' (N-mm)		19.77347	19.69394	19.75328
SS1' (psi)		7.99699	8.21946	8.16782
SS2 (psi)		1.38054	2.56990	1.79180
Variance		0.01173	0.00013	0.00017
Stiffness Coefficients (MPa)				
Sub1	C11s1	114999	112820	115000
	C22s1	26816.5	27238.6	26811.7
	C13s1	5832.7	5495.4	5832.7
	C44s1	1900	2347.64	1900
	C55s1	1901.87	3732.87	3328.59
Sub2	C11s2	95847.9	95847.9	95847.9
	C22s2	35847.9	35847.9	35847.9
	C13s2	5664.11	5664.11	5664.11
	C44s2	3550	3550	3550
	C55s2	3550	3550	3550

5. Verification Issues

In order to demonstrate the advantages of our proposed approach, two issues are verified here. The first issue to verify is whether the use of response surface models instead of the high fidelity analysis programs has significantly improved the computational efficiency of thick laminated composite structure design with a reasonable sacrifice in accuracy. For the simplistic structure model considered in this example, the proposed method requires 177 experiments (function evaluations) to generate the response surface models for both Level 1 and Level 2, which take approximately twelve hours on a Sparc 20 UNIX platform. Nevertheless, robust design using original finite element analysis would require more than 1000 function evaluations at each level of optimization, if three random levels are chosen for the noise factor. The total time for robust design optimization at two levels would take more than 136 hours on the same computer platform. This benefit will be even more significant for more complicated composite structures that require higher computational resource for each simulation and robust design problems with more noise factors. The proposed approach will provide more significant time savings in testing different optimization scenarios, since in those cases the response surface model representing the global design space behavior could be reused even though the optimization scenarios may change. The use of response surface models may also smooth the structure behavior and therefore increases the chance of fast convergence. In terms of accuracy, for the example problem, the optimization solutions obtained using second-order response surface models are identical to those using ABAQUS finite element analysis. We note that the number of functional evaluations will grow very rapidly if the number of design variables increases. To reduce the additional computational effort caused by the increased dimensionality and to better model the higher degrees of nonlinearities or irregularity in the structural behavior, alternative model fitting techniques such as neural networks (Smith, 1993; Hajela and Berke, 1992) and the Kriging method (Matheron, 1963) can be used to improve the accuracy.

The second issue to verify is the benefit of the robust design approach to structure optimization. In our verification test, the minimized variations of performance through robust design are compared with the performance variations of the conventional optimization solutions (without robust design considerations) assuming that the deviation of the noise factor (bone factor) is the same in both cases. In Level 1, the variance of energy E for the solution without robust design considerations is greater than the variance obtained for robust design (0.51359 vs. 0.2706). In Level 2, the variance of interfacial shear stress SS2 is 0.21614 versus 0.00017 for the same comparison. In both levels, the solutions with robust design considerations are much less sensitive to the deviation of bone factors. This indicates that the same principle can be used to reduce the impact of other parameter deviations in laminated composite structure design.

The decomposition of such a complex problem provides three primary benefits for efficient analysis. First, it substantially decreases the number of design variables in a given optimization process. Second, it overcomes the difficulties associated with convergence. For example, the maximization of strain energy of the calcar and the minimization of interfacial shear stress are two conflicting objectives. By separating them into different optimization levels, and using the first level's results as the second level's constraints, it overcomes the convergence difficulty which must be faced if they were put together in a multi-objective optimization. Third, it decreases the chances of obtaining too many local minimums thus interfering with finding the global minimum. The decomposition thus provides a very good design optimization scheme to solve the original design problem with substantial benefits.

6. Conclusions

In this paper, a systematic and affordable approach is proposed for the design of thick composite structures. Our approach integrates the principles of the Robust Concept Exploration Method (RCEM) for designing complex engineering systems and the hierarchical multi-level optimization procedure for managing the complexity of composite structure optimization. In this process, response surface models were employed to replace the expensive computer analysis required by direct structural optimization. By observing the regression coefficients and grid plots, the approximation models proved to be accurate enough to replace the comprehensive computer analyses required. This process, in coordination with DOE and RSM, proved to enhance the computational efficiency associated with the thick laminated composite structure design with a reasonable sacrifice in accuracy. The computational time for optimization with approximations was much smaller than the time required for optimization using original high fidelity analysis. At each level of the process, robust design solutions were generated with quality considerations by minimizing the effects of variation of the bone factor as well as optimizing the structural behavior. The benefits of using the robust design approach were verified by testing the variance of performance when the design variables were set to the values from the solution without robust design considerations (Section 5). It shows that the robust design solution is applicable for a range of bone factors, thereby eliminating the need to design components for a specific individual.

As for the effectiveness of the multi-level optimization scheme, our proposed scheme capitalizes on the computational efficiency associated with the global-local analysis method and with the reasonably small number of design variables to provide an efficient way to optimize a structure having three-dimensional geometry and loading, complex material properties, and multiple objectives.

Although our proposed approach is illustrated for the design of a simplified model of a laminated femoral component for hip joint replacement, the same procedure can be readily applied for robust optimization in other complex thick composite structures.

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