A PROCEDURE FOR ROBUST DESIGN: MINIMIZING VARIATIONS CAUSED BY NOISE FACTORS AND CONTROL FACTORS

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ABSTRACT

In this paper, we introduce a small variation to current approaches broadly called Taguchi Robust Design Methods. In these methods, there are two broad categories of problems associated with simultaneously minimizing performance variations and bringing the mean on target, namely,

Type I - minimizing variations in performance caused by variations in noise factors (uncontrollable parameters).

Type II - minimizing variations in performance caused by variations in control factor (design variables).

In this paper, we introduce a variation to the existing approaches to solve both types of problems. This variation embodies the integration of the Response Surface Methodology (RSM) with the compromise Decision Support Problem (DSP). Our approach is especially useful for design problems where there are no closed-form solutions and system performance is computationally expensive to evaluate. The design of a solar powered irrigation system is used as an example.

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1. **Our Frame of Reference**

The fundamental principle in robust design is to improve the quality of a product by minimizing the effects of variation without eliminating these causes. There are two broad categories of problems associated with simultaneously minimizing performance variations and bringing the mean on target, these categories are based on the source of variation:

- **Type I** - minimizing variations in performance caused by variations in noise factors (uncontrollable parameters).
- **Type II** - minimizing variations in performance caused by variations in control factor (design variables).

Taguchi's "parameter design concept" was developed for Type I applications (Phadke, 1989). In this method, basically a two-part orthogonal array is used for experimental design using the signal-to-noise-ratio as an optimization criterion. Although Taguchi's contribution to making optimization user friendly for the engineers who have very little training in optimization methods is well recognized (Nair, 1992), there are certain assumptions and limitations associated with his methods. For example, use of the Taguchi method will not yield an accurate solution for design problems that embody highly nonlinear behavior. In addition, the Taguchi method has been criticized by the statistical community (for example, Box, 1988, Nair, 1992, Tsui, 1992). Tsui, 1992, argues that many of Taguchi’s statistical methods, e.g., orthogonal arrays, linear graphs and accumulation analysis, are not statistically efficient. Box, 1988, points out that there are various mathematical difficulties/requirements associated with the use of signal-to-noise-ratio. Welch, et al., 1990, propose combining control and noise factors into a single array thus modeling the response rather than expected loss, and approximating a prediction model for loss based on the fitted-response model. This response-model approach is further developed by Shoemaker, et al., 1991. These proposed modifications
to the Taguchi method, however, still involve a single performance metric. In our opinion, since there are multiple objectives to be satisfied in design it follows that there must be multiple aspects to quality. This opinion is reinforced by some preliminary insight we gained using quality engineering techniques to reduce the number of the trajectory simulations of a LifeSat space vehicle (Lautenschlagar, et al., 1993) and to achieve a concurrent concept selection and system synthesis of a solar powered irrigation system (Chen, et al., 1994). Related to this view, Otto and Antonsson, 1991, argue the necessity of incorporating constraints in robust design; Parkinson, et al., 1993, propose to include feasibility robustness as an important robust design category, in which constraint feasibility under variation is insured.

Nonlinear programming methods have been used for both Type I and II applications. Ramakrishnana and Rao, 1991, formulate the robust design problem as a nonlinear optimization problem with Taguchi's loss function as the objective. They emphasize the numerical evaluation of total quality loss but fall short in addressing the nature of variations. Sundaresan, et al., 1993, incorporate a Sensitivity Index (SI) in the optimization procedure to determine a “robust optimum”. We find it is difficult to determine weighting factors for target performance and variance in the SI function. Sundaresan and his colleagues use concepts from the design of experiments to approximate the variation in performance at each iteration. However, this is computationally inefficient for large, analysis-intensive and computationally-intensive problems.

Our aim, in this paper, is to introduce a variation to the approaches suggested by Taguchi and further developed by others to accomplish robust design. Providing the foundation for our work is the synthesis of ideas from (a) the Taguchi method, (b) statistical experimentation methods, and (c) nonlinear programming methods. The outcome of this
synthesis is embodied in the integration of the Response Surface Methodology (RSM) with the compromise Decision Support Problem (DSP) (see Figure 1). The RSM is a collection of statistical techniques which support the design of experiments and the creation of a response model (Khuri and Cornell, 1987). The compromise DSP is a multiobjective mathematical construct (Mistree, et al., 1993) which enables a designer to determine values of design variables which satisfy a set of constraints to achieve a set of goals. The objective in a compromise DSP is to minimize the deviations of different goals from target values using lexicographic minimization (Ignizio, 1985).

- INSERT FIGURE 1 HERE -

**Figure 1. Integration of Response Surface Methodology with the Compromise DSP**

Using the terminology of quality engineering, response surface models are developed for the system performance as functions of control factors (system variables) and noise factors (system parameters with variation) over the region of interest which is defined by the bounds of design variables. Based on a response surface model, the mean and variance of a response are derived as functions of control factors according to the sources of variation (either the variation of noise factors, control factors, or both). The explicit forms of these equations are then passed to the compromise DSP where they are used as either constraints or goals. Instead of using a single criterion and minimizing the signal-to-noise ratio, an important aspect of this work is to permit a designer to independently bring the mean on target and minimize the variation around this target. This is accomplished by modeling these objectives separately.
In this paper\textsuperscript{5}, we first discuss the two sources of variation in robust design. We then show the steps of integrating the RSM and the compromise DSP for robust design. The solar powered irrigation system is used as an example to demonstrate our approach.

II A DESCRIPTION OF OUR APPROACH

Robust Design to Reduce Variation from Two Types of Source

The concepts behind the robust design applications associated with the two kinds of sources of variation are illustrated in Figure 2. On the left-hand side of Figure 2, we use a P-diagram (Phadke, 1987) to represent different types of parameters in robust design, their relationships with the whole system, and thus the differences in sources of variation in response for Type I and Type II robust design applications. Control factors ($x$) are parameters which can be specified freely by a designer; noise factors ($z$) are parameters that are not under a designer’s control; and signal factors ($M$) are the intended values for the response ($y$) of a product/process. When $M$ is a function rather than a specific value, robust design is a dynamic problem. In dynamic problems, the objective is to achieve robust performance over a range of values for the signal factors. Our approach to this situation is presented in (Chen, et al., 1996a).

In this paper, we assume that the signal factors have fixed values. In Type I applications, the variance in response is caused by variations in the noise factors. Type II robust design is different from Type I in that its input does not include a noise factor. The variation in performance is caused solely by variations in control factors or design variables in the region $\pm \Delta x$.

Figure 2. A Comparison of Two Types of Robust Design

On the right hand side of Figure 2 we present a schematic of the different concepts behind the two types of robust design. Taguchi’s parameter design is a Type I method and is highlighted in the upper right block of Figure 2. In the Taguchi method, a designer adjusts control factors, $x$, to dampen the variations caused by the noise factor, $z$. The two curves represent performance variation as a function of a noise factor when $x$ is at two different levels, $x = a$ and $x = b$. If the design objective is to achieve a performance as close as possible to the target, $M$, the designs at both levels are acceptable because their means are the target $M$. However, introducing robustness, when $x = a$, the performance varies significantly with the variations of noise factor, $z$, however when $x = b$, the performance varies much less. Therefore, $x = b$ is a better than $x = a$ as a design solution because it dampens the effect of the noise factors.

The concept behind robust design associated with Type II variation is represented in the lower right block of Figure 2. For purposes of illustration, assume that performance is a function of only one variable, $x$. Generally, in this type of robust design, to reduce the variation of response caused by variations of design variables, instead of seeking the optimum value, a designer is interested in identifying the flat part of a curve near the performance target. If the objective is to move the performance function towards target $M$ and if a robust design is not sought, then obviously the point $x = \mu_{opt}$ is chosen. However, for a robust design, $x = \mu_{robust}$ is a better choice. If design variables vary within the region $\pm \Delta x$ of their means, the resulting variation of response of the design at $x = \mu_{robust}$ is much smaller than that at $x = \mu_{opt}$, while the means of the response at two designs are close.
Although the concepts behind the two types of robust design are different, robust design is always concerned with aligning the peak of the bell shaped response distribution with the targeted quality, that is, bringing the mean on target, and making the bell shaped curve thinner by reducing variance. This makes it possible to develop a general procedure for robust design in which these objectives are modeled as separate goals in a compromise DSP.

A Robust Design Procedure

Our procedure that facilitates solving both types of robust design problems includes three major steps:

Step 1. Build response surface models to relate each response to all important control- and noise– factors using the Response Surface Methodology (RSM).

Step 2. Derive functions of mean and variance of the responses based on the type of robust design applications.

Step 3. Use the compromise DSP to find the robust design solution.

In Step 1, the response-model approach (Welch, et al., 1990: Shoemaker, et al., 1991) is applied to overcome the limitations of Taguchi's loss model approach. Using an integrated design analysis program as the simulation module, instead of applying Taguchi's inner- and outer-array approach, control and noise factors are put in a single array for computer experimentation. The RSM, is used to support the design of experiments and fitting a response model. In general, the response-model postulates a single, formal model of the type

\[ \hat{y} = f(x, z), \]

\[ (2.1) \]
where \( \hat{y} \) is the estimated response and \( x \) and \( z \) represent the settings of the control and noise variables. A second order model, which has linear terms, quadratic terms and interaction terms is the most frequently used surface model.

In Step 2, the mean and variability of the response are derived based on the response surface models (Eqn. 2.1). Taylor expansions can be used to approximate the variability of the response (performance). When the computational demand of using second-order Taylor expansion is large, first-order Taylor expansions are usually utilized as approximations. For Type I applications in which the variations of noise factors are the most important source of variation:

Mean of the response  
\[
\mu_{\hat{y}} = f(x, \mu_z) \quad (2.2)
\]

Variance of the response  
\[
\sigma_{\hat{y}}^2 = \sum_{i=1}^{k} \left( \frac{\partial f}{\partial z_i} \right)^2 \sigma_{z_i}^2, \quad (2.3)
\]

where \( \mu \) represents the mean values, \( k \) is the number of noise factors in the response model and \( \sigma_{z_i} \) is the standard deviation associated with each noise factor. Eqn. 2.3 is based on the first-order Taylor expansion (Phadke, 1989) and it is assumed that the noise variables are independent. In Type II robust design, i.e., when the variations of control factors are most important source of variation, to estimate the mean and the variability of the response, Eqns. (2.2) and (2.3) are modified:

Mean of the response  
\[
\mu_{\hat{y}} = f(x) \quad (2.4)
\]

Variance of the response  
\[
\sigma_{\hat{y}}^2 = \sum_{i=1}^{l} \left( \frac{\partial f}{\partial x_i} \right)^2 \sigma_{x_i}^2, \quad (2.5)
\]
Eqns. (2.3) or (2.5) can be combined to achieve a robust design if there are variations of both control and noise factors. Having built the approximating functions using the response surface model, we then come to Step 3 to solve the problem using the compromise DSP and these approximations rather than solving the problem directly using a computationally expensive analysis model. In this way, the variance of performance can be predicted rapidly instead of running multiple experiments at each iteration of the solution. Using the response-model approach, robust design for either type of application can be achieved by minimizing the variance Eqn. (2.3) or (2.5) and by bringing the mean to the target, Eqn (2.2) or (2.4). Because the classical RSM is restricted to unconstrained searching for a local optimum of a single response (or the variance of that response) a compromise Decision Support Problem is introduced to handle multiple aspects of quality and engineering constraints.

A typical structure for a compromise DSP is presented in Figure 1 and described in detail elsewhere (e.g., Mistree, et al., 1993). In the compromise DSP, each goal, $A_i$, has two associated deviation variables $d_i^-$ and $d_i^+$ which indicate the extent of the deviation from the target, $G_i$. To effect a solution, on the basis of preference, goals may be rank-ordered into priority levels using the lexicographic minimum (Ignizio 1985). Therefore, using the compromise DSP, it is possible to address individually the issues of maximizing the intensity of the signal on target and minimizing variation. These become separate goals in a multiobjective compromise DSP. This approach helps a designer focus on individual contributions to mean and variation and to identify parameters which affect the attainment of specific goals.

To study variations of constraints caused by variations of controllable or uncontrollable parameters, we use a worst case scenario instead of expected values of system performance for constraints. In the worst case scenario it is assumed that all variations of
system performance may occur simultaneously in the worst possible combinations of
design parameters and design variables. The use of the worst case scenario is more
conservative than the use of probabilistic constraints (Eggert and Mayne, 1993). When
information about the variations of design parameters or design variables is statistical,
instead of using \( g_j(x, z) \leq 0 \), the constraint becomes:

\[
E [ g_j(x, z)] + n\sigma_j \leq 0,
\]

where \( n \) is a constant that reflects the probability that the variation will be feasible. If the
variation in the constraint is normally distributed, \( n=3 \), means that the constraint will be
satisfied 99.87 percent of the time. The constraint variation \( \sigma_j \) can be derived based on
Eqns. 2.3 and 2.5. When the variation information is not statistical, \( g_j(x, z) \leq 0 \) becomes

\[
E [ g_j(x, z)] + \Delta g_j \leq 0,
\]

where \( \Delta g_j \) represents the performance variation which can be evaluated using the
following approximation:

\[
\Delta g_j = \sum_{i=1}^{l} \left| \frac{\partial g_j}{\partial x_i} \Delta x_i \right| + \sum_{i=1}^{k} \left| \frac{\partial g_j}{\partial z_i} \Delta z_i \right|
\]

where \( \Delta z_i \) is the variation of a noise parameter and \( \Delta x_i \), the variation of a sensitive design
variable (control factor); \( k \) and \( l \) are the numbers of noise parameters and sensitive design
variables, respectively. Overall, the compromise DSP provides a generic approach to
achieve robust design by enabling a designer to find values of control factors \( (x) \) to
achieve a mean of performance (Eqns. 2.2 and 2.4) which is as close as possible to the
targets \( (M) \) and to minimize variations (Eqns. 2.3 and 2.5) around the targets, subject to
feasibility robustness related to constraints (Eqns. 2.6 and 2.7). The solar powered irrigation system is used next to illustrate our approach.

III THE DESIGN OF A SOLAR POWERED IRRIGATION SYSTEM

Previously, we have used the solar powered irrigation system as an example to show the application of Taguchi techniques to concurrent subsystem embodiment and system synthesis (Chen, et al., 1994). We used inner- and outer- arrays for experiments and the signal-to-noise ratio as the measurement and found that there are many limitations of the Taguchi's approach, Section 1. We found it was difficult to use this method when making tradeoffs among multiple quality characteristics and incorporating constraints and factor interactions. In this paper, using the same problem, we show how the method proposed in this paper can be used to overcome these difficulties. The layout of the solar-powered irrigation system is shown in Figure 3. It is assumed that parabolic trough N-S tracking is used for solar collection and with water as the working fluid. The aim in the preliminary system design is to determine:

- maximum operating pressure, Rankine Cycle, \( X_A \) (MPA)
- maximum operating temperature, Rankine Cycle, \( X_B \) (°K)
- maximum temperature drop in the solar collectors, \( X_C \) (°K)
- working fluid flow rate, \( X_D \) (kg/s)

The design must meet the specifications:

- the target for the pumped load (power output) is 20 kW
- overall efficiency must be maximized toward a target value of 20%
- economic benefits must be maximized with a target of $150,000.
The design also must satisfy thermal system requirements, i.e., the maximum temperature in the cycle has to be smaller than the stagnation temperature at all times. (The stagnation temperature cannot be less than the maximum temperature corresponding to the maximum pressure). In this work, we combine thermodynamic property prediction software (Shamsundar, 1989) with an economic analysis routine (Bascaran, 1990) for our computer simulation module. Simulation inputs include definitions for components, system parameters involved in system synthesis and the operating environment. System performances, e.g., cycle efficiency, power output, total efficiency and economic benefits are generated. An important aspect of thermal system design is to reduce the variance in system performance caused by the variation of an uncontrollable operating environment (noise factors). In our case, the noise factors are ambient temperature and level of insolation. Type I robust design can be applied. Another important application of robust design is to reduce the variance of performance caused by variations in control factors considering that there may be adjustments to these factors in the later stages of design. In other words, it is necessary to find a flat region of design space rather than the optimum. We demonstrate how these two types of robust design can be formulated and solved.

The first step is to classify the parameters as control factors, noise factors, signal factors or responses. The ranges of these parameters and their relationship to the whole solar powered irrigation system are shown in Figure 4. There are four controllable design parameters, \(X_A\), \(X_B\), \(X_C\), and \(X_D\), representing decisions on thermal cycle design. Two noise factors (uncontrollable parameters, \(Z_a\) and \(Z_b\)) are associated with the unstable operating environment. Three signal factors define targets for the desired performance. Among the four responses, three of them (Power, Efficiency and Savings) are considered
as objectives and stagnation temperature is introduced as the constraint in the compromise DSP. After classifying parameters, it is necessary to fit a surface model for each of the four responses over the space formed by control factors, $X_A$, $X_B$, $X_C$, and $X_D$, and noise factors, $Z_a$ and $Z_b$.

**- INSERT FIGURE 4 HERE -**

*Figure 4. Classification of Design Parameters*

Using the RSM, there is always a tradeoff between the number of experiments used and the accuracy of the estimated model. Studies show that the response-model approach relies on the adequacy of simple models over a small parameter range. A common strategy is to use sequential experimentation, i.e., perform low order screening experiments first over the whole design space to identify appropriate regions of interest, and then build a higher order response model over reduced regions, probably also using a reduced number of factors. Here we assume the reduced parameter ranges are $X_A$ [2.0-3.0] MPA, $X_B$ [450-520] °K, $X_C$ [520-800] °K, and $X_D$ [0.01-0.045] kg/s. The ranges for the noise factors $Z_a$ and $Z_b$ are [293-303] °K and [800-1000] W/m$^2$ respectively as before.

From low-order screening experiments, we also notice that noise factor $Z_b$ and its associated interactions have no effect on the power and economic benefits while control factor $X_D$ and associated interactions do not influence the behavior of efficiency and stagnation temperature. Thus Central Composite Design (CCD) with 43 experiments for 5 factors can be used to construct quadratic response surfaces for each response (Montgomery, 1991). The CCD is composed of a $2^5$ (32) factorial design, 10 star points and 1 center point. The suggested value for $\alpha$, a parameter varied to meet conditions of equal precision of estimation in all directions, is $2.378$. Based on the results of the
experiment, the least squares method from regression analysis is used to fit a quadratic surface model:

\[ f(x_1, \ldots, x_n) = \beta_0 + \beta_1 x_1 + \ldots + \beta_n x_n + \gamma_1 x_1^2 + \ldots + \gamma_n x_n^2 \\
+ \beta_{12} x_1 x_2 + \ldots + \beta_{n,n-1} x_{n-1} x_n \]

A summary of the response surface models for the four system performances is presented in Table 1. The variables \(X_A, X_B, X_C, X_D, Z_a\) and \(Z_b\) which are used in the equations are all normalized between -1 and 1. One benefit of using normalized variables is that the total response variance can be calculated easily when there are several sources of variance. Based on the coefficients of the response model, a designer can see clearly the significance - or contributions - of different terms, that is, linear, quadratic and interaction terms. The functions in Table 1 are the reduced models with some trivial effects ignored, e.g., those items with very small coefficients. If, after normalization, the contribution of an item is less than 1%, it is considered trivial. The sequence of items is arranged in descending order of significance. In this case there are interaction factors and second-order factors from the three response surface models of Power, Efficiency and Savings (Eqns. 3.1 – 3.3), while the model for the stagnation temperature is a linear function (Eqn. 3.4).

| - INSERT TABLE 1 HERE - |

Table 1. Response Models for the System Performances

The normalized quadratic surface model can also help to identify potential control factors for reducing the noise effects by examining the control-noise interactions. For example, to reduce the variance of power caused by the noise factors \(Z_a\), the potential adjustment factors are \(X_A, X_B\) and \(X_D\). Further, using \(X_D\) is most effective because the coefficient of
XDZ_a is greater. Based on the response models, we can formulate compromise DSPs for Type I and Type II robust design applications.

**Compromise DSP Type I** Reducing the variance of system performance caused by the variation of the uncontrollable operating environment is associated with Type I variation, Table 2. Using the compromise DSP, the problem becomes how to choose X_A, X_B, X_C and X_D to reduce the influence from Z_a and Z_b while keeping the performance as close as possible to the targets.

- INSERT TABLE 2 HERE -

Table 2. The Compromise DSP Formulation I

Eqn. 3.5 in Table 2 represents the constraint on stagnation temperature. To handle multiple objectives simultaneously, each objective is modeled as a goal. As we choose to address separately the issues of bringing the mean to target and minimizing the deviation, for three system performance targets, there will be six goals, Eqns. 3.6 to 3.11. Tradeoffs are implemented by minimizing the total deviation function, Eqn. 3.12. When the objective needs to be maximized (or minimized), deviation variable d_i^- (or d_i^+) is used in the deviation function. When the objective is to achieve as close as possible to the target, (d_i^- + d_i^+) is used in the deviation function. In Eqn. 3.12, f_i through f_6 in Eqn. 3.12 represent different priority levels assigned to the six goals. Initially, all goals are at the same priority level and assigned equal weights. The deviation function becomes:

\[
Z = 0.167(d_1^- + d_1^+) + 0.167(d_2^+) + 0.167(d_3^-) \\
+ 0.167(d_4^+) + 0.167(d_5^-) + 0.167(d_6^+) \tag{3.13}
\]
Further information about the formulation and solution of compromise DSPs is available in Mistree, et al, 1993. Variances of performance are derived based on the first order Taylor expansion. Eqns. (3.5-3.11) can be expanded using the response surface models in Table 1, using Eqn. (3.1), Eqn. (3.6) becomes:

\[
(22.500 + 14.300X_D + 1.000X_B + 0.965X_A - 0.668X_A^2 + 0.637X_B X_D + 0.297X_A X_B - 0.064X_B^2)/T_{Powmean} + d_1^- - d_1^+ = 1. \tag{3.14}
\]

Similarly Eqn. (3.7) can be expanded:

\[
(-0.74 - 0.47X_D + 0.045X_A - 0.027X_B)^2 + d_2^- - d_2^+ = 0 \tag{3.15}
\]

In the compromise DSP, to calculate the performance variance, instead of using \(\sum_{i=1}^{k} \left( \frac{\partial f}{\partial z_i} \right)^2 \), only \(1/9 \sum_{i=1}^{k} \left( \frac{\partial f}{\partial z_i} \right)^2 \) is required. This is because when there are several noise factors, \(z_i\), after normalization, \(\sigma_{z_i}\) has the same value if the standard deviation is proportional to the range of design variables. This is equal to \(1/3\) assuming that the deviation follows a normal distribution. With respect to the goal of minimizing variance, it is desirable that the target approach zero. However, it is not always advisable to use zero as a target because tremendous resources are often required to reduce variance to zero. In this work, we first calculate a reasonable minimum value for the variance of a response without considering any other goals and use this value as the target for the variance goal. For Power and Efficiency zero becomes the target. The target for variance of economic benefits is \(1.778E6 \ ($2\)\).

The compromise DSP in Table 1 is solved using the DSIDES software (Mistree, et al., 1993a). After decoding the design variables into natural values, the results are:
At this solution point, the values of mean and variance of each performance are as follows with the total deviation function value, Z, equal to 0.03843:

\[
\begin{align*}
\text{Pow}_{\text{mean}} &= 19.0324 \text{ (kW)} & \text{Pow}_{\text{var}} &= 0.0363 \text{ (kW}^2) \\
\text{Eff}_{\text{mean}} &= 19.25\% & \text{Eff}_{\text{var}} &= 2.6084\times10^{-6} \\
\text{Sav}_{\text{mean}} &= 133781 \text{ ($)} & \text{Sav}_{\text{var}} &= 1.777\times10^6 \text{ ($}^2) \\
\end{align*}
\]

When using the response surface models for robust design, it is always important to conduct confirmation tests to check the accuracy of results. As explicit analytical equations are not available for this problem, to confirm the adequacy of the response model in predicting the mean and variance of system performance, random simulations are used. In Table 3, the results are compared to those from 100 random simulations and 500 random simulations. For these simulations, the values of the control factors are fixed at their solution point and the values of the noise factors vary within the given range. It can be noted that the estimations for the mean of power, mean of efficiency and mean of savings are quite accurate. For variance, in column 2 of Table 3, the estimated values are provided when assuming the noise factors are normally distributed or uniformly distributed. Random simulations yield values which are close to those obtained when assuming the noise factors are uniformly distributed. Note that the accuracy is satisfactory.

- INSERT TABLE 3 HERE -

Table 3. Confirmation of the Results

When considering multiple aspects of quality, designers may have different preferences for whether it is more important to bring the mean on target or reduce variation. In the
compromise DSP, different design scenarios can be modeled by assigning goals different priority levels. In Table 4, the results of three different preemptive formulations of deviation function are given.

- INSERT TABLE 4 HERE -

Table 4. Studies on Goal Priorities for Mean and Variance

A general observation from Table 4 is that the values of \( X_A \) and \( X_C \) stay the same for all scenarios while \( X_B \) jumps between its upper and lower bounds and \( X_D \) varies for different conditions. Checking the response surface models in Table 1, \( X_D \) is found to be the most important scaling factor as its linear effects are significant for both power and savings. To reduce the variance of power and savings a smaller \( X_D \) is preferred. This is the reason why \( X_D \) approaches its lower bound in Scenario III in which minimizing the variances is given the highest priority. However, \( X_D \) cannot be too small in order to bring the means on their targets. This is the reason why the values of \( X_D \) move up under Scenarios I and II in which bringing the mean on target is highly considered. Among them, the result of scenario I represent a tradeoff between bringing the mean on target and minimizing variance. The change of the values of \( X_B \) is mainly due to interactions with \( X_D \). It can also be observed from Table 1, that it is possible to reduce the variance of power and efficiency to very small values; however, it is impossible for the variance to reach zero.

Compromise DSP Formulation II Given that \( X_A, X_B, X_C \) and \( X_D \) are system-level design variables, adjustments are possible after the design at the component level is finalized. Type II robust design reduces the variance of performance due to variations of control factors. Suppose that within the design parameter range, the possible variation for each variable is constant, after normalization, say \( \Delta X_A = \Delta X_B = \Delta X_C = \Delta X_D = \pm 0.2 \), then
the compromise DSP for robust design in this new situation is given in Table 5. We assume that noise factors \( Z_a \) and \( Z_b \) become constants at their means \( Z_a = 298^\circ \text{K} \) and \( Z_b = 900 \text{W/m}^2 \). Eqn. 3.16 in Table 5 is the constraint on stagnation temperature. The goals are listed in Eqns. 3.17 to 3.22. Eqn. 3.23 is the total deviation function.

- INSERT TABLE 5 HERE -

Table 5. The Compromise DSP Formulation II – Robust Design Against Variation of the Control Factors

The target for each variance goal is the best possible value obtained without considering other goals. Similar to verifications of formulation I, we use three different deviation functions to test solutions for different scenarios, Table 6. Based on the results, the following observations are made:

- Comparing the deviation function values at different priority levels for scenarios II and III, it is noted that there are conflicts between the goals of bringing the mean on target and minimizing the variation. The goals can be better achieved when they are put at a higher priority level; e.g., for bringing the means on target, they are better achieved (0.0286 compared to 0.0484) when this is given the first priority as it is in Scenario II. A similar situation happens in minimizing the variance. Using the Archimedean function (Scenario I), a trade-off is made.

- The results for \( X_B \) and \( X_C \) stay the same for the different scenarios while \( X_D \) changes a little. Looking at the equations of means and variances, this is reasonable because a higher value of \( X_A \) is favorable for bringing the means on their targets. Thus, in Scenario II, in which the goal of bringing the mean on target is given a high priority, \( X_A \) moves to its upper bound.
• Under all the design scenarios, the goal for minimizing the variance associated with the power can always be achieved; the goal of bringing the mean of savings on its target is never achieved, and the rest of the goals are sometimes achieved, but sometimes underachieved.

- INSERT TABLE 6 HERE -
Table 6. Results of Type II under Different Scenarios

These observations match the mathematical structure of the response model, e.g., for the second observation, the equation of efficiency indicates that the larger $X_A$, the easier it is to bring it on target. However, this contradicts the notion of reducing variance of the other responses. If variance reduction is assigned a higher weight, an appropriate trade-off results.

Comparison of the Results Obtained by Using Our Approach and the Taguchi Method

As demonstrated by Types I and II applications our approach has the advantages of considering multiple quality aspects and incorporating feasibility robustness which the Taguchi method does not permit. The question now becomes *for a single quality performance and unconstrained robust design problem, will our approach be able to offer better, or at least equivalent solutions, compared to the Taguchi method?* The answer is yes. In Table 7 there is a comparison of the results obtained by using our approach and by using the Taguchi method. The robust design objective is to reach the Power target value 20 kW and to minimize the variance while there are variations in
noise factors. Five hundred random simulations are used to simulate the variation of noise factors, ambient temperature and insolation level.

From Table 7, it is noted that majority of the solutions (design variables) obtained by using the Taguchi method are at the bounds or factor levels of the design variables while the solutions obtained using our approach all lie between the bounds of design variables. The procedure for using the Taguchi method for the power irrigation system is presented in Chen, et al. 1994, and is not repeated here. With respect to bringing the mean on target and minimizing the variance, our solutions are slightly better than those obtained by using the Taguchi method. This is reasonable, because the responses of this problem are dominated by linear effects (Table 1). Taguchi's signal-to-noise ratio is a valid approach for a robust problem if the linear effects are additive. This indicates that the author's approach is consistent with Taguchi's robust design concepts.

IV CLOSURE

In our opinion, the integration of Response Surface Methodology with the compromise DSP appears to have several advantages:

- Engineering constraints can be introduced and there is great flexibility for studying tradeoffs among multiple design objectives.
- The interaction effects and nonlinear effects can be considered using the response surface model, e.g., quadratic surfaces. Compared to Taguchi’s linear model approach, this approach yields more accurate results.
- Response surface models can serve as fast analysis modules for different types of robust design application. By introducing a parameter as a variable in the response surface model, we extend our scope of study.
- The compromise DSP addresses individually the issues of bringing the mean on target and reducing variation. This provides designers more flexibility to make
decisions based on different robust design criteria and helps them focus on major hurdles and make improvements.

It is important to note the limitations of the method presented here. First, the goal and constraint functions are approximated using the methods of statistical design of experiments and specifically quadratic models. Although the approximations have been proved satisfactory for a wide variety of engineering problems (Chen 1995), there are cases in which the performance is highly nonlinear and second-order model is not good enough even within a reduced region. It is also important to note the assumptions for using the equations presented in this paper. For example, a linear Taylor expansion assumes the variation will be small and a statistical distribution of the variation is normal.

Robust design procedures can be applied to various kinds of engineering problems. In this paper we have illustrated our approach using a thermal system. Other examples, include aircraft (Chen, et al. 1996b) and aircraft engines (Koch et al., 1996).

ACKNOWLEDGMENTS

We are grateful to Professor N. Shamsunder, University of Houston, for providing the thermodynamic software. Wei Chen was supported by The Woodruff School of Mechanical Engineering, Georgia Tech during her graduate study. We gratefully acknowledge NSF grant DDM-93-96503 and NASA Grant NAG-1-1564.

REFERENCES


Shamsundar, N., 1989, "University of Houston Thermodynamic Properties Software," Department of Mechanical Engineering, University of Houston.


Find
System Variables x
Deviation Variables $d_1^-, d_1^+$

Satisfy
Constraints $g_i(x) \leq 0$
Goals
$A_i(x) + d_i^- - d_i^+ = G_i$
Bounds $x_{min} \leq x \leq x_{max}$

Minimize
Deviation Function
$Z = [f_1(d_i, d_i^+), ..., f_k(d_i, d_i^+)]$

Response Models for Constraints/Goals
$y = f(x, z)$
Mean
$\mu_y = f(\mu_x, \mu_z)$
Deviation
$\sigma_y^2 = \sum_{i=1} \left( \frac{\partial f}{\partial z_i} \right)^2 \sigma_{\tilde{z}_i}^2$

or
$\sigma_y^2 = \sum_{i=1} \left( \frac{\partial f}{\partial x_i} \right)^2 \sigma_{\tilde{x}_i}^2$

Figure 1. Integration of the Response Surface Method with the Compromise DSP
Figure 2. A Comparison of Two Types of Robust Design
Figure 3. Solar Powered Irrigation System
Solar Powered Irrigation System

Control Factors, X
- Maximum pressure $X_A [0.3–3.0]$ MPA
- Maximum temperature $X_B [450–520]$ °K
- Maximum temperature collector $X_C [520–800]$ °K
- Working fluid flowrate $X_D [0.01–0.2]$ kg/s

Signal Factors, M
- Power: 20 kW (nonimal the best)
- Overall efficiency 20% (bigger the better)
- Yearly savings $150,000$ (bigger the better)

Noise Factors, Z
- Ambient temperature $Za [293–303]$ °K
- Level of insolation $Zb [800–1000]$ W/m²

Response, Y
- Power output
- Overall efficiency
- Yearly savings
- Stagnation temperature

Figure 4. Classification of Design Parameters
Table 1. Response Models for System Performances

<table>
<thead>
<tr>
<th>Response Model</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Power (objective) kW</strong></td>
<td>( = 22.500 + 14.300X_D + 1.000X_B + 0.965X_A - 0.740Z_a + 0.668X_A^2 + 0.637X_BX_D + 0.591X_AX_D - 0.470X_DZ_a + 0.297X_A \times X_B - 0.064X_B^2 + 0.045X_AZ_a - 0.027X_BZ_a ) (3.1)</td>
</tr>
<tr>
<td><strong>Efficiency (objective)</strong></td>
<td>( = 0.17600 + 0.01210X_A - 0.00945X_C - 0.00426X_A^2 + 0.00413Z_b - 0.00374Z_a - 0.00142X_B - 0.00105X_CZ_b ) (3.2)</td>
</tr>
<tr>
<td><strong>Savings (objective) $</strong></td>
<td>( = 158000.00 + 100000.00X_D + 7030.00X_B + 6760.00X_A - 5180.00Z_a - 4680.00X_A^2 + 4460.00X_BX_D + 4140.00X_AX_D - 3290.00X_DZ_a + 2080.00X_AX_B - 444.00X_B^2 - 188.00X_BZ_a + 313.00X_AZ_a ) (3.3)</td>
</tr>
<tr>
<td><strong>Tstag °K (constraint)</strong></td>
<td>( = 1873 + 174.8Z_b + 5Z_a ) (3.4)</td>
</tr>
</tbody>
</table>
Table 2. The Compromise DSP Formulation I – Robust Design to Reduce the Effects of Variations in Noise Factors

**Given**
- Response model of Power, Overall Efficiency, Savings and Stagnation Temperature as functions of XA, XB, XC, XD, Za and Zb
- The mean and variance of noise factors, \( \mu_{Za} = 0, \mu_{Zb} = 0, \sigma_{Za} = 1/3, \sigma_{Zb} = 1/3 \) (all normalized).
- Target for Power and its variance, \( T_{Pow mean} = 20 \text{ kW} \) (the nominal the better), \( T_{Pow var} = 0 \).
- Target for Efficiency and its variance, \( T_{Eff mean} = 20\% \) (the bigger the better), \( T_{Eff var} = 0 \).
- Target for Savings and its variance, \( T_{Sav mean} = $150000 \) (the bigger the better), \( T_{Sav var} = 1.778E6(\$^2) \)

**Find**
- The values of control factors
  - \( X_A \), Normalized cycle maximum pressure
  - \( X_B \), Normalized cycle maximum temperature
  - \( X_C \), Normalized collector maximum temperature
  - \( X_D \), Normalized working fluid flow rate
- The values of deviation variables \( d_i^- , d_i^+ \) \( (i =1, 6) \)

**Subject to**
- The constraint:
  
  \[
  E[T_{stag}(X_A, X_B, X_C, X_D, Z_a, Z_b)] \geq \mu_{Za}\cdot \frac{\partial T_{stag}}{\partial Z_a}\cdot 3\sigma_{Za} + \mu_{Zb}\cdot \frac{\partial T_{stag}}{\partial Z_b}\cdot 3\sigma_{Zb} \quad (3.5)
  \]

  where \( E \) represents the statistical *expected value* of a function

- The goals:
  
  Achieve as closely as possible the target value for the power mean
  \[
  \text{Pow}(X_A, X_B, X_C, X_D, \mu_{Z_a}, \mu_{Z_b})/T_{Pow mean} + d_1^- - d_1^+ = 1, \quad (3.6)
  \]
  Minimize the variance of power
  \[
  [(\partial \text{Pow}/\partial Z_a)^2 + (\partial \text{Pow}/\partial Z_b)^2]^{1/2} + d_2^- - d_2^+ = 0, \quad (3.7)
  \]
  Maximize mean of overall efficiency to the target
  \[
  \text{Eff}(X_A, X_B, X_C, X_D, \mu_{Z_a}, \mu_{Z_b})/T_{Eff mean} + d_3^- - d_3^+ = 1, \quad (3.8)
  \]
  Minimize the variance of overall efficiency
  \[
  [(\partial \text{Eff}/\partial Z_a)^2 + (\partial \text{Eff}/\partial Z_b)^2]^{1/2} + d_4^- - d_4^+ = 0, \quad (3.9)
  \]
  Maximize mean of savings to the target
  \[
  \text{Sav}(X_A, X_B, X_C, X_D, \mu_{Z_a}, \mu_{Z_b})/T_{Sav mean} + d_5^- - d_5^+ = 1, \quad (3.10)
  \]
  Minimize the variance of savings
  \[
  [(\partial \text{Sav}/\partial Z_a)^2 + (\partial \text{Sav}/\partial Z_b)^2]^{1/2} + d_6^- - d_6^+ = 0, \quad (3.11)
  \]

- Bounds on the design variables:
  - \(-1 \leq X_A \leq 1\)
  - \(-1 \leq X_B \leq 1\)
  - \(-1 \leq X_C \leq 1\)
  - \(-1 \leq X_D \leq 1\)

  \( d_i^+ , d_i^- \geq 0 \)

**Objective**

Minimize the total objective function
\[ Z = [ f_1(d_1^-,d_1^+), f_2(d_2^+), f_3(d_3^-), f_4(d_4^+), f_5(d_5^-), f_6(d_6^+) ] \]  (3.12)
Table 3. Confirmation of the Results

<table>
<thead>
<tr>
<th></th>
<th>From the Second-Order Response Model</th>
<th>From 100 Random Simulations</th>
<th>From 500 Random Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of Power (kW)</td>
<td>19.032</td>
<td>19.284</td>
<td>19.263</td>
</tr>
<tr>
<td>Mean of Efficiency</td>
<td>19.32%</td>
<td>19.37%</td>
<td>19.32%</td>
</tr>
<tr>
<td>Mean of Savings ($)</td>
<td>133781</td>
<td>135044</td>
<td>134898</td>
</tr>
<tr>
<td>Variance of Power (kW²)</td>
<td>0.0363*/0.101**</td>
<td>0.091</td>
<td>0.121</td>
</tr>
<tr>
<td>Variance of Efficiency</td>
<td>2.6084E-6*/7.824E-6**</td>
<td>7.163E-06</td>
<td>9.175E-06</td>
</tr>
<tr>
<td>Variance of Savings ($)²</td>
<td>1.777E+6*/5.331E+6**</td>
<td>4.469E+06</td>
<td>5.928E+06</td>
</tr>
</tbody>
</table>

* assuming the noise factors are normally distributed  
** assuming the noise factors are uniformly distributed

Table 4. Studies on Goal Priorities for Mean and Variance (Type I)

<table>
<thead>
<tr>
<th></th>
<th>Scenario I</th>
<th>Scenario II</th>
<th>Scenario III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Formulation of Deviation Function</td>
<td>Level 1–All six goals weighted equally</td>
<td>Level 1–Mean</td>
</tr>
<tr>
<td></td>
<td>System Variables</td>
<td>Level 2–Variance</td>
<td>Level 2–Mean</td>
</tr>
<tr>
<td></td>
<td>XA (MPA)</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>XB (°K)</td>
<td>520</td>
<td>450</td>
</tr>
<tr>
<td></td>
<td>XC (°K)</td>
<td>520</td>
<td>520</td>
</tr>
<tr>
<td></td>
<td>XD (kg/s)</td>
<td>0.02189</td>
<td>0.02575</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>19.0324</td>
<td>19.9975</td>
</tr>
<tr>
<td></td>
<td>Power (Target 20 kW)</td>
<td>19.25%</td>
<td>19.53%</td>
</tr>
<tr>
<td></td>
<td>Efficiency (Target 20%)</td>
<td>133781</td>
<td>140463</td>
</tr>
<tr>
<td></td>
<td>Savings (Target $150,000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Power (Target 0)</td>
<td>0.0363</td>
<td>0.0429</td>
</tr>
<tr>
<td></td>
<td>Efficiency (Target 0)</td>
<td>2.6084E-06</td>
<td>2.6084E-06</td>
</tr>
<tr>
<td></td>
<td>Savings (Target 1.778E+06)</td>
<td>1.777E+06</td>
<td>2.102E+06</td>
</tr>
<tr>
<td></td>
<td>Deviation Function Value Z</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>level 1</td>
<td>0.03843</td>
<td>0.02870</td>
</tr>
<tr>
<td></td>
<td>level 2</td>
<td>0.07429</td>
<td>0.43822</td>
</tr>
</tbody>
</table>
Table 5. The Compromise DSP Formulation II – Robust Design to Reduce the Impact of Variations in Control Factors

Given
- Response model of Power, Overall Efficiency, Savings and Stagnation Temperature as functions of $X_A$, $X_B$, $X_C$, $X_D$, $Z_a$ and $Z_b$.
- $Z_a = Z_b = 0$
- $\Delta X_A = \Delta X_B = \Delta X_C = \Delta X_D = 0.2$
- Target for Power and its variance, $TPow_{mean} = 20$ kW (the nominal the better), $TPow_{var} = (3)^2$ kW$^2$, (equivalent to $\Delta Y = 3$ kW)
- Target for Efficiency and its variance, $TEff_{mean} = 20\%$ (the bigger the better), $TEff_{var} = 0$.
- Target for Savings and its variance, $TSav_{mean} = $150000 (the bigger the better), $TSav_{var} = 3.61E8$ $\$^2$, (equivalent to $\Delta Y = 19000 \$)$

Find
- The values of control factors
  - $X_A$, Normalized cycle maximum pressure
  - $X_B$, Normalized cycle maximum temperature
  - $X_C$, Normalized collector maximum temperature
  - $X_D$, Normalized working fluid flow rate
- The values of deviation variables $d_i^-$, $d_i^+$ ($i = 1, 6$)

Subject to
- The constraints:
  - The stagnation temperature cannot be less than the maximum temperature in cycle
    $E[T_{stag}(X_A, X_B, X_C, X_D, Z_a, Z_b)] - [\frac{\partial T_{stag}}{\partial X_A}]^* \Delta X_A - [\frac{\partial T_{stag}}{\partial X_B}]^* \Delta X_B$
    $\geq (X_B + 1* \Delta X_B)*500$ (3.16)
- The goals:
  - Achieve the mean of power as close as possible to the target
    $Pow(X_A, X_B, X_C, X_D, Z_a, Z_b)/TPow_{mean} + d_1^- - d_1^+ = 1$, (3.17)
  - Minimize the variation of power
    $[\frac{\partial Pow}{\partial X_A}]^2 + [\frac{\partial Pow}{\partial X_B}]^2 + [\frac{\partial Pow}{\partial X_C}]^2 + [\frac{\partial Pow}{\partial X_D}]^2/TPow_{var} + d_2^- - d_2^+ = 0$, (3.18)
  - Maximize mean of overall efficiency to the target
    $Eff(X_A, X_B, X_C, X_D, Z_a, Z_b)/TEff_{mean} + d_3^- - d_3^+ = 1$, (3.19)
  - Minimize the variance of overall efficiency
    $[\frac{\partial Eff}{\partial X_A}]^2 + [\frac{\partial Eff}{\partial X_B}]^2 + [\frac{\partial Eff}{\partial X_C}]^2 + [\frac{\partial Eff}{\partial X_D}]^2 + d_4^- - d_4^+ = 0$, (3.20)
  - Maximize mean of savings to the target
    $Sav(X_A, X_B, X_C, X_D, Z_a, Z_b)/TSav_{mean} + d_5^- - d_5^+ = 1$, (3.21)
  - Minimize the variance of savings
    $[\frac{\partial Sav}{\partial X_A}]^2 + [\frac{\partial Sav}{\partial X_B}]^2 + [\frac{\partial Sav}{\partial X_C}]^2 + [\frac{\partial Sav}{\partial X_D}]^2/TSav_{var} + d_6^- - d_6^+ = 0$, (3.22)
- Bounds on the design variables
  - $-1 \leq X_A \leq 1$
  - $-1 \leq X_B \leq 1$
  - $-1 \leq X_C \leq 1$
  - $-1 \leq X_D \leq 1$
  - $d_i^+ - d_i^- = 0$, with $d_i^+, d_i^- \geq 0$
Objective
Minimize the total deviation function

\[ Z = [ f_1(d_1^- + d_1^+), f_2(d_2^+), f_3(d_3^-), f_4(d_4^+), f_5(d_5^-), f_6(d_6^+)] \]  

(3.23)
Table 6. Results from Type II Robust Design for Different Scenarios

<table>
<thead>
<tr>
<th>Formulation of Deviation Function</th>
<th>Scenario I</th>
<th>Scenario II</th>
<th>Scenario III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level 1–All six goals weighted equally</td>
<td>Level 1–Mean Level 2–Variance</td>
<td>Level 1–Variance Level 2–Mean</td>
</tr>
<tr>
<td><strong>System variables</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>XA (MPA)</td>
<td>1.7047</td>
<td>3.0</td>
<td>1.7019</td>
</tr>
<tr>
<td>XB (°K)</td>
<td>450.0</td>
<td>450.0</td>
<td>450.0</td>
</tr>
<tr>
<td>XC (°K)</td>
<td>520.0</td>
<td>520.0</td>
<td>520.0</td>
</tr>
<tr>
<td>XD (kg/s)</td>
<td>0.02587</td>
<td>0.02575</td>
<td>0.02594</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power (Target 20 kW)</td>
<td>19.9519</td>
<td>19.9999</td>
<td>20.0009</td>
</tr>
<tr>
<td>Efficiency (Target 20%)</td>
<td>18.34%</td>
<td>19.53%</td>
<td>18.34%</td>
</tr>
<tr>
<td>Savings (Target $150,000)</td>
<td>140143</td>
<td>140480</td>
<td>140486</td>
</tr>
<tr>
<td><strong>Variance</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power (Target 9 kW²)</td>
<td>7.3839</td>
<td>8.2255</td>
<td>7.3826</td>
</tr>
<tr>
<td>Efficiency (Target 0)</td>
<td>1.2388E-05</td>
<td>4.8619E-06</td>
<td>1.2415E-05</td>
</tr>
<tr>
<td>Savings (Target 3.61E+08 $²)</td>
<td>3.61E+08</td>
<td>4.02E+08</td>
<td>3.61E+08</td>
</tr>
<tr>
<td><strong>Deviation Function Value Z</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level I</td>
<td>0.0253</td>
<td>0.0286</td>
<td>0.4097E-05</td>
</tr>
<tr>
<td>Level II</td>
<td>0.0377</td>
<td>0.0484</td>
<td></td>
</tr>
</tbody>
</table>

Table 7. Comparison of the Taguchi Method and our Approach for and Unconstrained Problem and Single Quality Performance

<table>
<thead>
<tr>
<th>System Variables</th>
<th>Taguchi Method</th>
<th>Our Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>XA (MPA)</td>
<td>3.0</td>
<td>2.5975</td>
</tr>
<tr>
<td>XB (°K)</td>
<td>520</td>
<td>506.39</td>
</tr>
<tr>
<td>XC (°K)</td>
<td>520</td>
<td>574.44</td>
</tr>
<tr>
<td>XD (kg/s)</td>
<td>0.0288</td>
<td>0.0234</td>
</tr>
<tr>
<td><strong>Mean of Power (kW)</strong></td>
<td>20.073</td>
<td>20.054</td>
</tr>
<tr>
<td><strong>Stan. Dev. of Power (kW²)</strong></td>
<td>0.38101</td>
<td>0.34189</td>
</tr>
</tbody>
</table>