Propagation and Management of Uncertainties in Simulation-Based Collaborative Systems Design

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1. Abstract

Simulation-based design has become an inherent part of multidisciplinary design as simulation tools provide designers with a flexible and computationally cheaper means to explore the interrelationships among various disciplines. Complications arise when the simulation programs may have deviations associated with input parameters (*external uncertainties*) as well as *internal uncertainties* due to the inaccuracies of the simulation tools or system models. These uncertainties will have a great influence on design negotiations between various disciplines and may force designers to make conservative decisions. In this paper, an integrated methodology for the propagation and management (mitigation) of uncertainty is proposed. Two approaches, namely, the *extreme condition approach* and the *statistical approach,* are developed to propagate the uncertainties. Using the extreme condition approach, an interval of the output from a chain of simulations is obtained, while the statistical approach provides a statistical distribution of the output. An uncertainty mitigation strategy based on the principles of robust design is developed. The methodology is successfully verified using the case study of a four bar linkage design.

2. Keywords

Simulation-Based Design, Multidisciplinary, Uncertainty, Robust Design, Statistical Approach, Extreme Condition

3. Introduction

The advancements in Computer Aided Engineering (CAE) have resulted in the development of simulation tools that model the behavior of real world systems. These tools provide designers with flexible and cheap means to deal with complicated systems analysis and design under a multidisciplinary collaborative design environment. Collaborative systems design [1-4] usually involves the interaction of various systems connected by linking variables. In collaborative system design, each subsystem (subdiscipline) may have two types of uncertainty as follows:

- *external uncertainty*
  External uncertainty comes from the variability in model prediction arising from plausible alternatives for input values x (including both design parameters and design variables) [5,6]. It is also called "*input parameter uncertainty*". Examples include of the variability associated with loading, material properties, physical dimensions of parts, and operating conditions.
- *internal uncertainty*
  This type of uncertainty has two sources[7,8]. One is due to the limited information in estimating the characteristics of model parameters for a given, fixed model structure *called "model parameter uncertainty"*, and another type is in the model structure itself, including uncertainty in the validity of the assumptions underlying the model, referred to as "*model structure uncertainty*".

A critical issue is that the uncertainties of one discipline may propagate to another through linking variables and the final system output will have a culmination of the uncertainties of the individual disciplines. The presence of various uncertainties will have a great influence on design negotiations between various disciplines and may force designers to make conservative decisions. The primary issues that arise are - How should the uncertainties be propagated across the disciplines? How should we manage (mitigate) the uncertainties and make reliable decisions?

In this paper, an integrated methodology for the propagation and management of uncertainties is proposed. Two approaches, namely, the *extreme condition approach* and the *statistical approach,* are developed to propagate the uncertainties. An uncertainty mitigation strategy based on the principles of robust design is developed. The proposed method is illustrated by the design of a four bar linkage.

4. Propagation of Uncertainties

In this section, a simplified systems design model as illustrated in Fig. 1 is used to explain the proposed methodology. This simplified model has been selected to facilitate the discussions of the proposed methodology in a more explicit manner. It should be noted that the principles of the methodology are generic and valid for other categories of relationships between the system models. The simulation-based systems design model illustrated in Fig. 1 consists of a chain of two simulation models (from two different disciplines) connected to each other through *linking variables* represented by the vector *y*. The input to the simulation model I is the vector of the design variables — x_1 with uncertainty (*external uncertainties*, Δx_1 or described by distributions). Due to the *external uncertainty* and the *internal uncertainty*, modeled as $e_1(x_1)$, of simulation model I, the output vector $y = F_1\ (x_1) + e_1\ (x_1)$ will have deviations (Δy or described by distributions). For simulation model II, the inputs are the linking variable vector (*y*) and the design variable vector x_2. Because of the deviations existing in x_2, *y*, and the *internal uncertainty* $e_2(x_2, y)$ associated with simulation II, the final output vector $z = F_2\ (x_2, y) + e_2(x_2, y)$ also deviates (Δz or described by distributions).

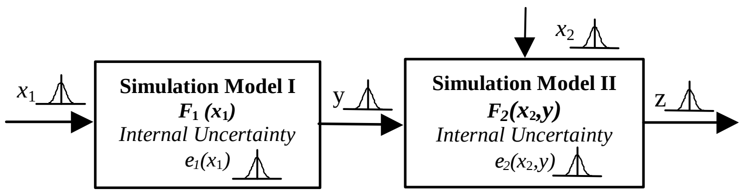

Figure 1 Simulation model chain

The Extreme Condition Approach for Uncertainty Propagation

The goal of the *extreme condition approach* is to obtain an interval of the *extremes* of the final output from a chain of simulation models. The term *extreme* is defined as the minimum or maximum value of the end performance (final output) corresponding to the given ranges of internal and external uncertainties. With this approach, the external uncertainties are characterized by the intervals $[x_1 - \Delta x_1, x_1 + \Delta x_1]$ and $[x_2 - \Delta x_2, x_2 + \Delta x_2]$. Optimizations are used to find the maximum and minimum (extremes) of the outputs from simulation model I and simulation model II. The flow chat of the proposed procedure is illustrated in Figure 2. The steps to obtain the range of output z, $[z^{\min}, z^{\max}]$, are developed as:

a) Minimize (Maximize) $F_1(x_1)$ and $e_1(x_1)$ by picking values from $[x_1 - \Delta x_1, x_1 + \Delta x_1]$ to obtain $F_1^{\min}(x_1)$ ($F_1^{\max}(x_1)$) and $e_1^{\min}(x_1)$ ($e_1^{\max}(x_1)$);

b) Obtain the interval $[y^{\min}, y^{\max}]$ with $y^{\min} = F_1^{\min}(x) + e_1^{\min}(x)$ and $y^{\max} = F_1^{\max}(x) + e_1^{\max}(x)$;

c) Minimize (Maximize) $F_2(x, y)$ and $e_2(x_2, y)$ by picking values from $[x_2 - \Delta x_2, x_2 + \Delta x_2]$ and $[y^{\min}, y^{\max}]$ to obtain $F_2^{\min}(x_2, y)$ ($F_2^{\max}(x_2, y)$) and $e_2^{\min}(x_2, y)$ ($e_2^{\max}(x_2, y)$);

d) Obtain the interval $[z^{\min}, z^{\max}]$ with $z^{\min} = F_2^{\min}(x_2, y) + e_2^{\min}(x_2, y)$ and $z^{\max} = F_2^{\max}(x_2, y) + e_2^{\max}(x_2, y)$.

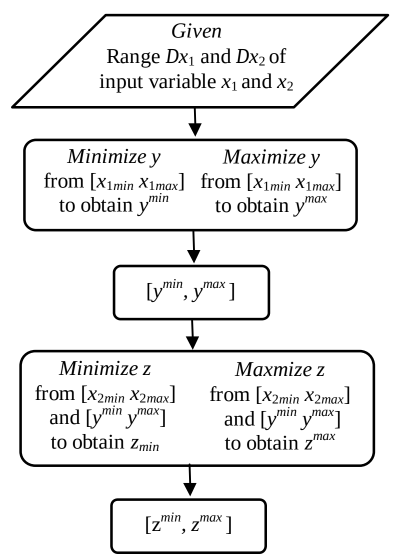

Figure 2 Procedure of the extreme condition approach

Based on the computed interval $[z^{\min}, z^{\max}]$, the nominal value of z can be calculated as

$$\bar{z} = \frac{1}{2}(z^{\min} + z^{\max}) \quad (1)$$

The deviation of z can be calculated as

$$\Delta z = z^{\max} - z^{\min} \tag{2}$$

This approach identifies the interval of a system output based on the given intervals of the system inputs.

<u>The Statistical Approach for Uncertainty Propagation</u>

The goal of the statistical approach is to obtain the complete probabilistic information of the final output from a chain of simulation models. The probabilistic information could be the cumulative distribution function (c.d.f) of the whole system output.

Assume the distributions of x_1 and x_2 follow probability density functions (p.d.f.) $f_1(x_1)$ and $f_2(x_2)$, and the internal uncertainties $e_1(x_1)$ and $e_2(x_2, y)$ are normally distributed. The mean values of these uncertainty distributions are $m_{e1}(x_1)$ and $m_{e2}(x_2, y)$ respectively, and the standard deviations of these uncertainties are $s_{e1}(x_1)$ and $s_{e2}(x_2, y)$ respectively. Monte Carlo simulation methods [9] are used to propagate the uncertainties through the simulation chain. The procedure to propagate the uncertainties is developed as:

a) Generate the samples of vectors x_1 and x_2 as simulation inputs based on p.d.f. $f_1(x_1)$ and $f_2(x_2)$
b) Calculate $F_1(x_1)$ and determine $m_{e1}(x_1)$ and $s_{e1}(x_1)$ of the internal uncertainty $e_1(x_1)$ for simulation model I;
c) Generate N samples of the internal uncertainty e_{1i} for simulation model I from the normal distribution $N(m_{e1}(x_1), s_{e1}(x_1))$ and obtain the samples of output $y_i = F_1(x_1) + e_{1i}$ for simulation model I;
d) For each y_i, calculate $F_2(x_2, y_i)$ and determine $m_2(x_2, y)$ and $s_2(x_2, y)$ of the internal uncertainty $e_2(x_2, y)$ for simulation model II;
e) Generate M samples of the internal uncertainty e_{2i} for simulation model II from the normal distribution $N(m_{e2}(x_2, y), s_{e2}(x_2, y))$ and obtain the samples of output $z_{ij} = F_2(x_2, y_i) + e_{2j}$ $\quad j = 1, \cdots, M$ for simulation model II;
f) Repeat steps a) ~ e) H times
g) Calculate the mean value m_z, the standard deviation s_z, or the p.d.f. of z by $H \times M \times N$ samples z_{ij}.

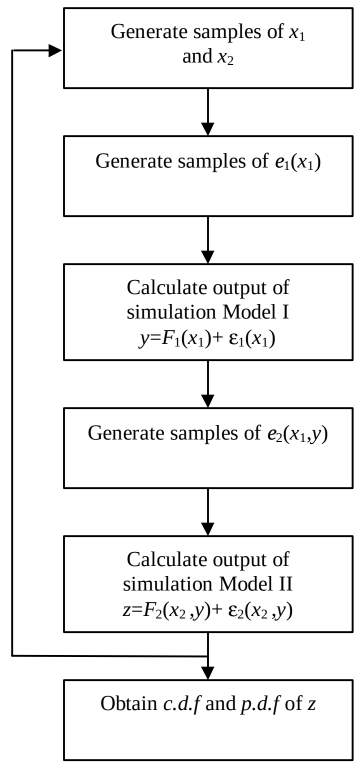

Figure 3 The process of Monte Carlo simulation

Figure 3 depicts the process of this simulation-based approach.

This approach identifies the distribution of the system output. based on the given p.d.f. of the inputs. Because it is based on the concept of Monte Carlo simulation, it often requires a large number of simulations.

5. Management of Uncertainties

To assist designers to make reliable design decisions under uncertainties, we integrate the proposed techniques of uncertainty propagation with the multidisciplinary optimization approach based on the principles of robust design, i.e. to extend the quality engineering concept to the mitigation of both external and internal uncertainties. From the viewpoint of robust design[10,11], the goal of a design is to make the system (or product) least sensitive to the uncertainty without eliminating the sources of uncertainty. The robust optimization objective is achieved by simultaneously "optimizing the mean performance" and "reducing the performance variation", subject to the robust constraints[12-14]. For the statistical approach, the robust design model can be formulated as:

Given:	Parameter and model uncertainties (distributions)
Find:	Robust design decisions (x)
Subject to:	System Constraints: $P[g(x) \le 0] \ge P_{\lim it}$
Objectives:	a. Optimize the mean of system attributes m_z b. Minimize the standard deviation of system attributes s_z

In the above model, $g(x)$ is the constraint function of system. The mean and variance of the system outputs (both objectives and constraints) could be obtained by the statistical approach for uncertainty propagation. Note that we have multiple objectives, i.e., both the mean value and variance of the system are expected to be minimized (here we assume optimizing the mean of a system attribute can be transformed into a minimization problem). The general form of the objective can be expressed as

$$\min \quad [m_z(x), s_z(x)]$$

Many existing multicriteria programming approach can be used to solve the above multiobjective optimization problem[12-17]. Note that the above constraints are expressed by the probabilistic formulation. $P[g(x) \le 0]$ is the probability of constraint satisfaction and it should be bigger than or equal to the defined probability $P_{\lim it}$. Other alternative forms can be used to express the constraints, such as the worst case analysis[12,13], the moment matching method[12,13] etc.

For the extreme condition approach, the robust design model can be formulated as:

Given:	Parameter and model uncertainties (ranges)
Find:	Robust design decisions (x)
Subject to:	System Constraints: $g(x) - k\nabla g(x)\Delta x \ge 0$
Objectives:	a. Optimize the mean of system attributes $\bar{z}$ b. Minimize the deviation of system attributes Δ_z

The above constraints are base on the worst case analysis where k is a constant which stands for the probability of constraint satisfaction [13] and $\nabla g(x)$ is the gradient of $g(x)$.

The integrated uncertainty mitigation strategy is shown in the Figure 4.

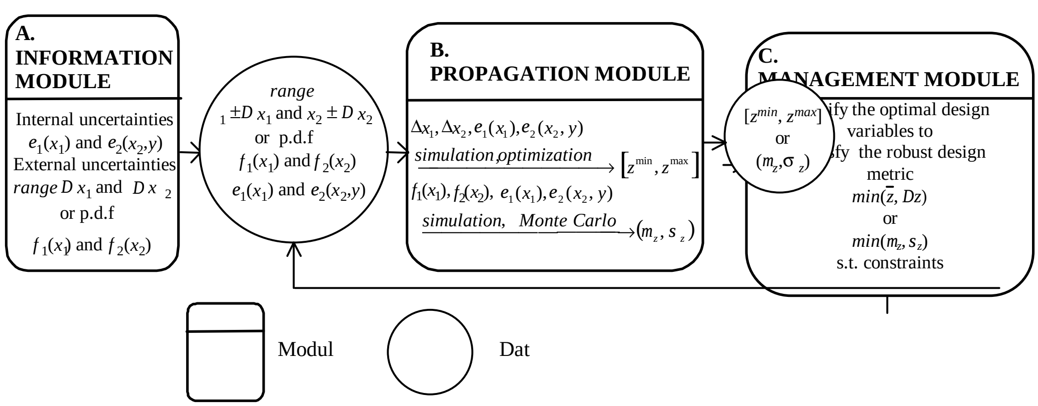

Figure 4 Integrated uncertainty mitigation strategy

6. Example—Mechanism Synthesis

The design of a crank-rocker four-bar linkage (See Fig. 5a) is used as a case study to illustrate the tangible effects of the proposed approach. The functional requirements are that when the input angles (crank angles relative to its initial position) are $j = 10^\circ$ and $j = 30^\circ$, the output angles (rocker angles) y are desired to be 123° and 125°, respectively. The length of link b, the length of rocket c, and the length of frame d are given as $b = 1.2$ mm, $c = 1.0$ mm, and d =

1.0mm. The minimum transmission angle must be bigger than 36°. The design variables are $x = [x_1, x_2]$, where $x_1 = j_0 =$ the initial angle of crank and $x_2 = a =$ the length of crank.

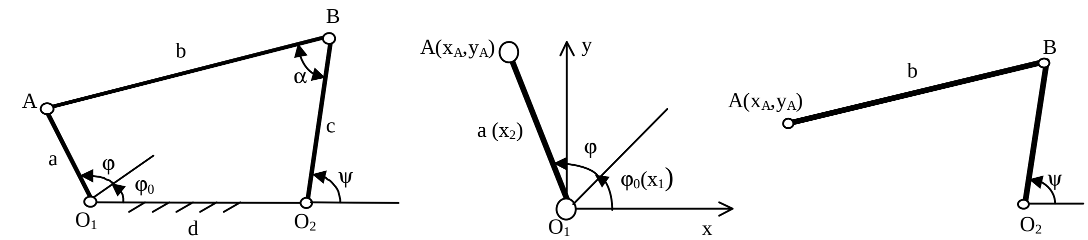

Figure 5a. Four-bar linkage Figure 5b. Crank subsystem Figure 5c. Link-Rocker subsystem

According to the theory of mechanism analysis analysis[18], the original four-bar system can be divided into two subsystems (1. crank subsystem, see Fig. 5b and 2. link-rocker subsystem, see Fig. 5c). For subsystem 1, the inputs are x_1, x_2 and the outputs are the coordinates of point $A(x_A, y_A)$. For sub-system 2, the inputs are x_2, and the coordinates of point $A(x_A, y_A)$, and the outputs are the angles of Rocker y_1, y_2, and the minimum transmission angle $a^{\min}$.

Response surface models are created based on the analytical models for both subsystems. These models can be considered as the simulation models in the Figure 1. For sub-system 1, the response surface models are $x_{A1} = f_{xA1}(x_1, x_2)$, $y_{A1} = f_{yA1}(x_1, x_2)$, $x_{A2} = f_{xA2}(x_1, x_2)$ and $y_{A2} = f_{yA2}(x_1, x_2)$. For sub-system 2, the response surface models are $j_1 = f_{y_1}(x_{A1}, y_{A1})$, $j_2 = f_{y_2}(x_{A2}, y_{A2})$ and $a^{\min} = f_{a_{\min}}(x_2)$. The mean values and standard deviations of the errors for each response surface model m_{xA1}, s_{xA1}, m_{yA1}, s_{yA1}, m_{xA2}, s_{xA2}, m_{yA2}, s_{yA2}, m_{y_1}, m_{y_2}, s_{y_1}, s_{y_2}, $m_{a_{\min}}$ and $s_{a_{\min}}$ are estimated using 100 random samples of the input variables. We will use these simplified models to design the mechanism with the consideration of uncertainties. In order to show the effect of the presented method, the result will be compared with those from the response surface models without consideration of uncertainties (conventional optimization).

For the statistical approach, the optimization model based on robust design concept can be stated as

$$Min\ F(x_1, x_2) = w_1 \sum_{i=1}^{2} (m_{y_i} - ?_1)^2 + w_2 \sum_{i=1}^{2} s_{y_i}^{\ 2}$$

$$\text{subject to } m_{a_{\min}} - k s_{a_{\min}} \geq 36^{\circ}$$

where w_1 and w_2 are weighting factors and $k = 2$ (This means with 97.72% probability, the constraint will be satisfied under the assumption that $a^{\min}$ is normally distributed). For the extreme condition approach, we have the similar optimization formulation. The results obtained from the proposed integrated uncertainty propagation and management approach are shown in Table 1. These results (listed under extreme condition approach and statistical approach) are compared to those from conventional optimization without consideration of uncertainties.

Table 1 Comparison of Design Results

Method	x_1 (°)	x_2 (mm)	y_1 (°)	y_2 (°)	$a^{\min}$ (°)	Deviation from targets(°)	
						Position 1	Position 2
Extreme Condition Approach	37.6378	0.289810	121.796	114.299	36.2434	-1.204	-0.701
Statistical Approach	35.3950	0.291867	122.685	114.934	36.1254	-0.315	-0.066
Conventional Optimization	36.0457	0.295998	122.269	114.488	35.8886	-0.731	-0.512

Note that the result from the statistical approach is better than the one from the conventional optimization approach, both in the achieved objective and the constraint. Even through the achieved objective based on the extreme condition approach is not as good as the one from the conventional optimization, the former generates a feasible solution while the later is infeasible since the minimum transmission angle is less than 36°.

7. Conclusions

An integrated methodology for the propagation and management of uncertainties in Simulation-Based Collaborative Systems Design is proposed in this paper. The *extreme condition approach* and the *statistical approach* are developed to propagate the uncertainties and they are integrated with the proposed uncertainty mitigation strategy based on the principles of robust design. It is shown through the example that the propagation and management of both external and internal uncertainties involved in simulation-based design can enable designers to make reliable decisions under uncertainties and risks. The proposed methodology is flexible and comprehensive with ample potential for its application in the area of multidisciplinary collaborative systems design. The concepts and principles presented in this paper can be expanded to more complicated systems.

The future work is targeted toward developing computationally efficient methods for uncertainty propagation and mitigation in a coupled multidisciplinary design environment where the performance prediction of one discipline may be the inputs of another discipline and vice versa. To decrease the computational effort in propagation of uncertainties by Monte Carlo simulations, we are developing methodology for fast and direct probabilistic evaluations of system performance based on the concepts of sensitivity analysis and reliability analysis.

8. Acknowledgement

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9. References

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