A Hierarchical Approach to Collaborative Multidisciplinary Robust Design

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Abstract
A hierarchical approach to robust design optimization considering the needs of robust performance from multiple disciplines is proposed in this paper. The approach extends the framework of Collaborative Optimization to perform robust design with multiple objectives from different disciplines under uncertainty. The proposed approach is demonstrated by examples and compared with the all-in-one approach.

Keywords: Robust Design, Hierarchical System, Uncertainty Analysis, Collaborative Optimization, Multiobjective Optimization

1. Introduction

The application of robust design under a multidisciplinary design optimization (MDO) environment has been gaining increasing attention in recent studies [1-8]. Though the usefulness of robust design is widely acknowledged for multidisciplinary design systems, its implementation is rare. The difficulties are associated with the complexity and computational burden for evaluating performance variations due to the randomness in a system, as well as the challenge of coordinating the needs from multiple disciplines, both in the mean and variance of performance. To ease the computational burden, some researchers utilize the technique of design of experiments (DOE) to replace the computationally expensive (analysis/optimization) models. Robust design is then performed based on the simplified models at the system level [2-5]. Other approaches [6-8], for example, system uncertainty analysis (SUA) and concurrent subsystem analysis (CSSUA) [7] we developed, improve the efficiency of multidisciplinary robust design by developing efficient uncertainty propagation techniques to evaluate the performance variations at the system level, taking into account the intrinsic relationships between subsystems analyses. In principle, all of these approaches belong to the "all-in-one" type in terms of how the tradeoff is made among multiple disciplines on robust performance. The robust design objective is formed for a single performance attribute either as the output of a particular subsystem or the combination of outputs from multiple subsystems, considering both the mean and variance aspects. The feasibility of the constraints from multiple disciplines (feasibility robustness) is simultaneously considered in a single model at the system level. The all-in-one approach is nonhierarchical in terms of the optimization framework for robust design.

Our interest in this paper is to investigate a hierarchical approach to robust optimization considering the needs of robust performance from multiple disciplines. We extend the framework of Collaborative Optimization [10, 11] to perform robust design with multiple objectives from different disciplines. For a hierarchical system, the information flows between a parent and children back and forth and there is no connection among children [9]. Hierarchical system approach was successful in formulating structural optimization and in solving very large MDO problems [9]. As an example of Hierarchical system, the Collaborative Optimization (CO), initially proposed for single objective MDO problems, hierarchically decomposes a complex system along discipline boundaries into a number of subsystems. The system level objective is optimized while system constraints are distributed in the subsystem level optimization where the objectives are to minimize the discrepancy between the interactive variables and the targets. It was then extended to the multiobjective collaborative optimization (MCO) in [12]. All the existing applications of CO are limited to deterministic optimization. It is our aim in this work to investigate the applicability of this framework when extended to multiobjective robust design under uncertainty. The proposed approach will be compared with the aforementioned all-in-one approach in terms of the efficiency in uncertainty analysis (evaluating performance variations and constraint feasibilities), and the optimization convergence behavior in robust design.

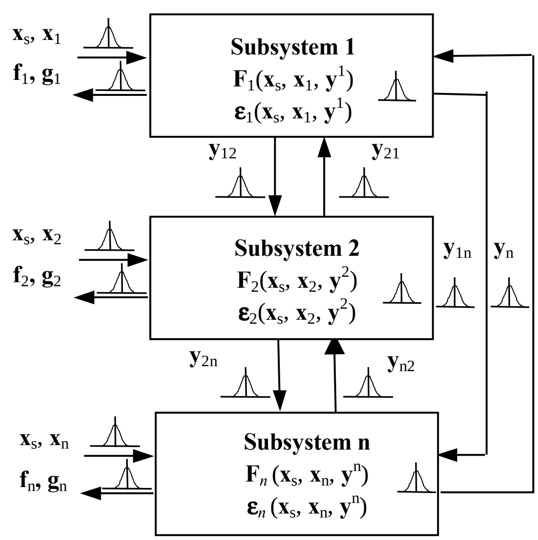

Figure 1 Coupled system

2. Problem Statement

Fig.1 shows the n-discipline system, where each box represents a simulation program that belongs to a discipline (or subsystem) for design evaluation. $\mathbf{x}_s$ are the input variables considered by more than one subsystem, also called sharing variables. $\mathbf{x}_i$ $(\mathrm{i}=1,\mathrm{n})$ are the input variables of subsystem i. Note that $\mathbf{x}_s$ and $\mathbf{x}_i$ are mutually exclusive sets of input variables. Uncertainties may be associated with $\mathbf{x}_s$ and $\mathbf{x}_i$ which can be expressed by probabilistic distributions. $\mathbf{y}_{ij}\,(i\neq j)$ are linking variables, which are those functional outputs calculated in subsystem i , at the same time, are required as inputs to subsystem j. For simplification of representation, we denote $\mathbf{y}_i=\{\mathbf{y}_{ij}\,|\, j=1,n, j\neq i\}$ as the set of linking variables generated as outputs from subsystem i and taken as inputs to the other subsystems and $\mathbf{y}^i=\{\mathbf{y}_1,\cdots,\mathbf{y}_{i-1},\mathbf{y}_{i+1},\cdots,\mathbf{y}_n\}$ as the set of linking variables generated as outputs from each of the subsystem except subsystem i and taken as inputs to subsystem i.

For subsystem i, based on the subsystem simulation model $\mathbf{F}_{yi}(\cdot)$ and the corresponding model error $\boldsymbol{\varepsilon}_{yi}(\cdot)$, the linking variables can be derived as:

$$\mathbf{y}_i=\mathbf{F}_{yi}(\mathbf{x}_s,\mathbf{x}_i,\mathbf{y}^i)+\boldsymbol{\varepsilon}_{yi}(\mathbf{x}_s,\mathbf{x}_i,\mathbf{y}^i) \tag{1}$$

Similarly, the general output of subsystem i, $\mathbf{z}_i(\mathrm{i}=1,\mathrm{n})$, can be derived as:

$$\mathbf{z}_i=\mathbf{F}_{zi}(\mathbf{x}_s,\mathbf{x}_i,\mathbf{y}^i)+\boldsymbol{\varepsilon}_{zi}(\mathbf{x}_s,\mathbf{x}_i,\mathbf{y}^i)\,. \tag{2}$$

where $\mathbf{z}_i$ consists of subsystem objective $\mathbf{f}_i$ and $\mathbf{g}_i$. The task of robust design in MDO at the system level can be stated as

Given: Standard deviations ($\boldsymbol{\sigma}_{\mathbf{x}}$, $\boldsymbol{\sigma}_{\mathbf{x}s}$) of design variables $\mathbf{x}_i$ and $\mathbf{x}_s$; means ($\boldsymbol{\mu}_{\varepsilon y}$, $\boldsymbol{\mu}_{\varepsilon f}$, $\boldsymbol{\mu}_{\varepsilon g}$) and standard deviations ($\boldsymbol{\sigma}_{\varepsilon y}$, $\boldsymbol{\sigma}_{\varepsilon f}$, $\boldsymbol{\sigma}_{\varepsilon g}$) of model uncertainty

Find: Robust design decisions ($\boldsymbol{\mu}_{\mathbf{x}}$ and $\boldsymbol{\mu}_{\mathbf{x}s}$)

Subject to: System constraints:

$\boldsymbol{\mu}_g+k\boldsymbol{\sigma}_g\leq 0$

Objectives: Optimize the mean of system attributes $\boldsymbol{\mu}_f$

Minimize the standard deviation of system attributes $\boldsymbol{\sigma}_f$

where $\mathbf{f}=[\mathbf{f}_1\quad \mathbf{f}_2\quad \cdots\quad \mathbf{f}_n]$, $\mathbf{g}=[\mathbf{g}_1\quad \mathbf{g}_2\quad \cdots,\quad \mathbf{g}_n]$, $\mathbf{y}=[\mathbf{y}_1\quad \mathbf{y}_2\quad \cdots\quad \mathbf{y}_n]$.

The above model corresponds to the all-in-one approach, where the vectors **f** and **g** are collections of objectives and constraints from multiple disciplines. To perform robust design, the variances of system output need to be evaluated in terms of means of input variables $\mathbf{x}_s$ and $\mathbf{x}_i$. In Ref. 7, the system uncertainty analysis (SUA) approach and the concurrent subsystem uncertainty analysis (CSSUA) approach are developed by authors to evaluate efficiently the variances of system output. The obtained information at the system level can be directly used in the all-in-one multidisciplinary robust design framework. Both methods were developed to reduce the amount of system level analysis. The idea of the SUA is to cut off the couplings between subsystems by solving the simultaneous linear equations of linking variables. First, the means of linking variables **y** are calculated by system level analysis and then the linking variables of each subsystem are linearized by Taylor expansions at the means of input variables in each subsystem. These linear equations of linking variables are assembled at the system level. After sovling the linking variables, the variances of system output can be evaluated based on Eq. 2. To completely avoid the system level analysis and to enhance parallization, the CSSUA uses optimization to solve the means of linking variables by maintaining the compatibilities between subsystems. In this research, our interest is to develop a hierarchical approach to multidisciplinary robust design that enables concurrent analysis and optimization within subsystems. This is proposed as an alternative approach to the all-in-one multidisciplinary robust design framework.

3. Hierarchical Robust Optimization Formulation

The proposed approach includes three levels of tasks, namely, system level robust optimization, subsystem level optimization, and subsystem level uncertainty analysis. System level optimization calls subsystem level optimization that calls subsystem level uncertainty analysis.

3.1 System level robust optimization

The system level robust optimization is used to minimize a system level multiobjective robust design function while satisfying compatibility constraints, which reflect coupling between subsystems. Many methods can be used to formulate the objective function that supports a multiobjective, collaborative decision making scheme. The system level performance **f** is a function of the subsystem level performance and its mean $\boldsymbol{\mu}_{\mathbf{f}}$ and standard deviation $\boldsymbol{\sigma}_{\mathbf{f}}$ are evaluated by subsystems. To fully match the random linking variables $\mathbf{y}$, their means $\boldsymbol{\mu}_{\mathbf{y}}$, variances $\boldsymbol{\sigma}^2_{\mathbf{y}}$, and covariance $\boldsymbol{\sigma}^2_{\mathbf{y}iyj}$ and $\boldsymbol{\sigma}^2_{\mathbf{yx}s}$ should be matched simultaneously. The system level robust optimization takes all these parameters and the means of sharing variable $\boldsymbol{\mu}_{\mathbf{x}s}$ as design variables. While the objective is to optimize the robustness of the system performance, the constraints are the equality constraints $d_i=0$ that match the target values of $\boldsymbol{\mu}_{\mathbf{y}}^0, \boldsymbol{\sigma}_{\mathbf{y}_i\mathbf{y}_j}^0, \boldsymbol{\sigma}_{\mathbf{y}_i\mathbf{x}_s}^0$ identified at the system level and those actual values $\boldsymbol{\mu}_{\mathbf{y}}, \boldsymbol{\sigma}_{\mathbf{y}iyj}, \boldsymbol{\sigma}_{\mathbf{y}jxs}$ generated from the subsystem analysis. The formulation of system level robust optimization is as follows

$$\begin{aligned}
&\mathbf{min} \quad \mathbf{F} = [\boldsymbol{\mu}_{\mathbf{f}}, \quad \boldsymbol{\sigma}_{\mathbf{f}}] \\
&s.t. \quad d_i = 0 \quad i = 1, n \\
&D.V.\, x^0 = [\boldsymbol{\mu}_{\mathbf{x}s}^0, \boldsymbol{\mu}_{\mathbf{y}}^0, \boldsymbol{\sigma}_{\mathbf{y}iyj}^0, \boldsymbol{\sigma}_{\mathbf{y}ixs}^0]
\end{aligned}$$

3.2 Subsystem level optimization

At the subsystem level, minimizing the difference between the target values of the means of sharing variables, variances and covariances of the linking variables from system level optimization and those actually generated through subsystems analyses becomes an objective. In the meantime, only constraints that belong to a particular discipline are considered in the subsystem level optimization. Therefore, in terms of the robust design considerations, only the feasibility robustness is considered at this level. The idea can be represented by the following optimization model:

$$\begin{aligned}
&d_i = (\boldsymbol{\mu}_{\mathbf{y}i} - \boldsymbol{\mu}_{\mathbf{y}i}^0)^2 + \sum_{j=1}^{n} (\boldsymbol{\sigma}_{\mathbf{y}iyj} - \boldsymbol{\sigma}_{\mathbf{y}iyj}^0)^2 + (\boldsymbol{\sigma}_{\mathbf{y}ixs} - \boldsymbol{\sigma}_{\mathbf{y}ixs}^0)^2, \\
&s.t. \quad \boldsymbol{\mu}_{\mathbf{g}i} + k\boldsymbol{\sigma}_{\mathbf{g}i} \le 0 \\
&D.V.\, \mathbf{x}^i = [\boldsymbol{\mu}_{xi}]
\end{aligned}$$

In the constraint formulation, k is a constant related to the probability of constraint satisfaction. For example, when follows the normal distribution, k=1 corresponds to the probability of 0.8413 and k=2 to the probability of 0.9972 [13].

3.3 Subsystem level uncertainty analysis

The means, variances, and covariances are all evaluated through subsystem level analyses. This task belongs to the so-called propagation of uncertainty effects which studies the impact of the uncertainty associated with simulation predictions on the analyses of individual subsystems as well as that of the whole system. The uncertainty analysis formulation is stated as

Given: mean values, variances and covariances of input variables for subsystem i

$\boldsymbol{\mu}_{\mathbf{x}_i}, \boldsymbol{\sigma}_{\mathbf{x}_i}^2, \boldsymbol{\mu}_{\mathbf{x}_{si}}^0, \boldsymbol{\sigma}_{\mathbf{x}_{si}}^{02}, \boldsymbol{\mu}_{\mathbf{y}^i}^0, \boldsymbol{\sigma}_{\mathbf{y}^i}^{02}, \boldsymbol{\sigma}_{\mathbf{y}^i\mathbf{x}_s}^{02}$

mean values and variances of error models

$\boldsymbol{\mu}_{\varepsilon y^i}, \boldsymbol{\sigma}_{\varepsilon y^i}^2, \boldsymbol{\mu}_{\varepsilon zi}, \boldsymbol{\sigma}_{\varepsilon zi}^2$

Find: mean values, variances and covariances of subsystem outputs

$\boldsymbol{\mu}_{\mathbf{f}i}, \boldsymbol{\sigma}_{\mathbf{f}i}^2, \boldsymbol{\mu}_{\mathbf{g}i}, \boldsymbol{\sigma}_{\mathbf{g}i}^2, \boldsymbol{\mu}_{\mathbf{y}i}, \boldsymbol{\sigma}_{\mathbf{y}i}^2, \boldsymbol{\sigma}_{\mathbf{y}ixs}^2$

The mean values of linking variables, objectives and constraints $\boldsymbol{\mu}_{\mathbf{f}i}, \boldsymbol{\mu}_{\mathbf{g}i}$, and $\boldsymbol{\mu}_{\mathbf{y}i}$ are approximated by Eqs. 1 and 2 at the mean values $\boldsymbol{\mu}_{\mathbf{x}i}, \boldsymbol{\mu}_{\mathbf{x}s}^0$ and $\boldsymbol{\mu}_{\mathbf{y}^i}^0$. ,

$$\boldsymbol{\mu}_{yi} = \mathbf{F}_{yi}(\boldsymbol{\mu}_{\mathbf{x}s}^0, \boldsymbol{\mu}_{xi}, \boldsymbol{\mu}_{\mathbf{y}^i}^0) + \boldsymbol{\mu}_{\varepsilon yi}, \tag{3}$$

$$\boldsymbol{\mu}_{zi} = \mathbf{F}_{zi}(\boldsymbol{\mu}_{\mathbf{x}s}^0, \boldsymbol{\mu}_{xi}, \boldsymbol{\mu}_{\mathbf{y}^i}^0) + \boldsymbol{\mu}_{\varepsilon zi}. \tag{4}$$

To obtain the variances and covariance $\boldsymbol{\sigma}_{\mathbf{f}}^2, \boldsymbol{\sigma}_{\mathbf{g}i}^2, \boldsymbol{\sigma}_{yi}^2$ and $\boldsymbol{\sigma}_{yixs}^2$, first, linking variables $\mathbf{y}_i$ and subsystem output $\mathbf{z}_i$ ($\mathbf{f}_i$ and $\mathbf{g}_i$) are linearized by the first-order Taylor approximation at the mean values $\boldsymbol{\mu}_{\mathbf{x}_i}, \boldsymbol{\mu}_{\mathbf{x}_{si}}^0$ and $\boldsymbol{\mu}_{\mathbf{y}}^0$ based on Eqs. 1 and 2 as

$$\Delta\mathbf{y}_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{\partial\mathbf{F}_{yi}}{\partial\mathbf{y}_j}\Delta\mathbf{y}_j + \frac{\partial\mathbf{F}_{yi}}{\partial\mathbf{x}_s}\Delta\mathbf{x}_s + \frac{\partial\mathbf{F}_{yi}}{\partial\mathbf{x}_i}\Delta\mathbf{x}_i + \Delta\boldsymbol{\varepsilon}_{yi} \tag{5}$$

$$\Delta\mathbf{z}_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{\partial\mathbf{F}_{zi}}{\partial\mathbf{y}_j}\Delta\mathbf{y}_j + \frac{\partial\mathbf{F}_{zi}}{\partial\mathbf{x}_s}\Delta\mathbf{x}_s + \frac{\partial\mathbf{F}_{zi}}{\partial\mathbf{x}_i}\Delta\mathbf{x}_i + \Delta\boldsymbol{\varepsilon}_{zi}. \tag{6}$$

Then, within each individual subsystem, the variances and covariance $\boldsymbol{\sigma}_{\mathbf{f}_i}^2, \boldsymbol{\sigma}_{\mathbf{g}_i}^2, \boldsymbol{\sigma}_{\mathbf{y}_i}^2$ and $\boldsymbol{\sigma}_{\mathbf{y}_i\mathbf{x}_{si}}^2$ are obtained based the approximation. Eqs (7) and (8) are expressions of variances.

$$\boldsymbol{\sigma}_{yi}^{*2} = \sum_{\substack{j=1 \\ j\neq i}}^{n} \left(\frac{\partial\mathbf{F}_{yi}}{\partial\mathbf{y}_j}\right)^2 \boldsymbol{\sigma}_{yj}^2 + \left(\frac{\partial\mathbf{F}_{yi}}{\partial\mathbf{x}_s}\right)^2 \boldsymbol{\sigma}_{xs}^2 + \left(\frac{\partial\mathbf{F}_{yi}}{\partial\mathbf{x}_i}\right)^2 \boldsymbol{\sigma}_{xi}^2 + \boldsymbol{\sigma}_{\varepsilon yi}^2, \tag{7}$$

$$\boldsymbol{\sigma}_{zi}^2 = \sum_{j=1}^{n}\left(\frac{\partial \mathbf{F}_{zi}}{\partial \mathbf{y}_j}\right)^2 \boldsymbol{\sigma}_{yj}^2 + \left(\frac{\partial \mathbf{F}_{zi}}{\partial \mathbf{x}_s}\right)^2 \boldsymbol{\sigma}_{xs}^2 + \left(\frac{\partial \mathbf{F}_{zi}}{\partial \mathbf{x}_i}\right)^2 \boldsymbol{\sigma}_{xi}^2 + \boldsymbol{\sigma}_{\varepsilon zi}^2 . \tag{8}$$

From the procedure, we see that compared to the all-in-one approach, all the optimizations and analyses are implemented only based on subsystem analyses and no system level analyses are involved. Therefore the subsystem level optimizations and subsystem analyses can be parallelized. For simplicity and efficiency, we may assume that all the linking variables and system output are probabilistically independent with each other and with the input variables. This will reduce the computational effort on evaluating covariances. Fig.2 shows the flowchart of the proposed hierarchical approach without consideration of correlations.

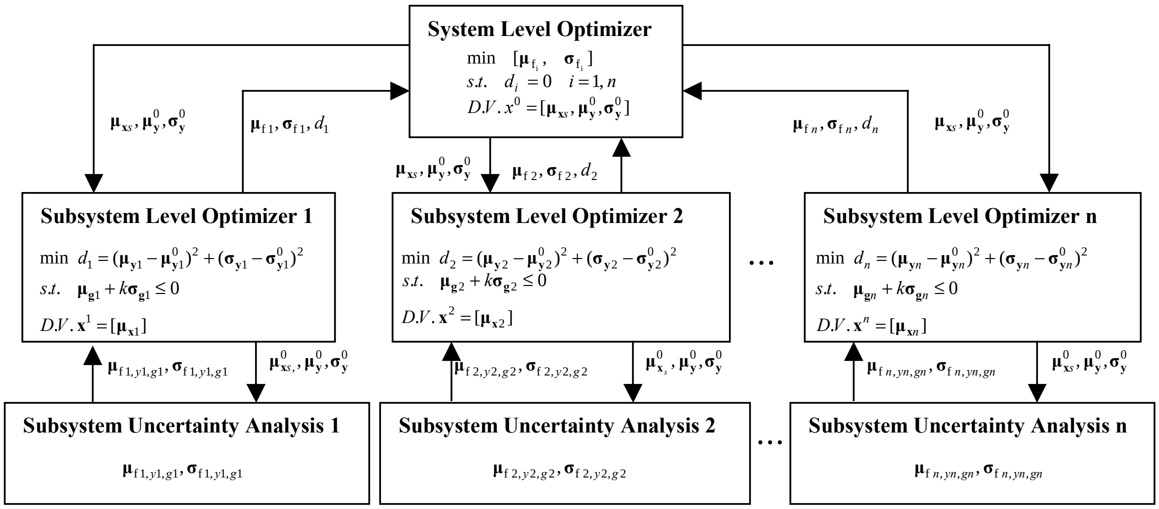

Figure 2 A Hierarchical Approach to Collaborative Multidisciplinary Robust Design

4. Example

The system shown in Fig. 3 has two subsystems. The functional relationships are represented as:

Subsystem 1

$\mathbf{x}_s = \{x_1\}$, $\mathbf{x}_1 = \{x_2, x_3\}$, $\mathbf{y}_1 = \mathbf{y}_{12} = \{y_{12}\}$, $\mathbf{z}_1 = \{z_1\}$, $\boldsymbol{\varepsilon}_{y1}(\mathbf{x}_s, \mathbf{x}_1, \mathbf{y}_1) = \{\varepsilon_{y12}(\mathbf{x}_s, \mathbf{x}_1, \mathbf{y}_2)\}$, $\boldsymbol{\varepsilon}_{z1}(\mathbf{x}_s, \mathbf{x}_1, \mathbf{y}_1) = \{\varepsilon_{z1}(\mathbf{x}_s, \mathbf{x}_1, \mathbf{y}_1)\}$,

$\mathbf{F}_{y12}(\mathbf{x}_s, \mathbf{x}_1, \mathbf{y}_1) = \{F_{y12}(\mathbf{x}_s, \mathbf{x}_1, \mathbf{y}_1)\} = x_1^2 + 2x_2 - x_3 + 2\sqrt{y_{21}}$, $\mathbf{F}_{z1}(\mathbf{x}_s, \mathbf{x}_1, \mathbf{y}_1) = \{F_{z1}(\mathbf{x}_s, \mathbf{x}_1, \mathbf{y}_1)\} = x_1^2 + 2x_2 + x_3 + x_2 e^{-y_{21}}$

Subsystem 2

$\mathbf{x}_s = \{x_1\}$, $\mathbf{x}_2 = \{x_4, x_5\}$, $\mathbf{y}_2 = \mathbf{y}_{21} = \{y_{21}\}$, $\mathbf{z}_2 = \{z_2\}$, $\boldsymbol{\varepsilon}_{y2}(\mathbf{x}_s, \mathbf{x}_2, \mathbf{y}_2) = \{\varepsilon_{y21}(\mathbf{x}_s, \mathbf{x}_2, \mathbf{y}_1)\}$, $\boldsymbol{\varepsilon}_{z2}(\mathbf{x}_s, \mathbf{x}_2, \mathbf{y}_2) = \{\varepsilon_{z2}(\mathbf{x}_s, \mathbf{x}_2, \mathbf{y}_2)\}$,

$\mathbf{F}_{y21}(\mathbf{x}_s, \mathbf{x}_2, \mathbf{y}_2) = \{F_{y21}(\mathbf{x}_s, \mathbf{x}_2, \mathbf{y}_2)\} = x_1 x_4 + x_4^2 + x_5 + y_{12}$, $\mathbf{F}_{z2}(\mathbf{x}_s, \mathbf{x}_2, \mathbf{y}_2) = \{F_{z2}\} = \sqrt{x_1} + x_4 + x_5(0.4 y_{12})$

The parameter and model error uncertainties are represented by distributions as defined in Table 1.

Table 1. Parameter and Model Uncertainties

	μ	σ	Distribution
x_1	μ_{x1}	0.1	Normal
x_2	μ_{x2}	0.1	Normal
x_3	μ_{x3}	0.1	Normal
x_4	μ_{x4}	0.1	Normal
x_5	μ_{x5}	0.1	Normal
ε_{y12}	0	$0.1\mu_{y12}$	Normal
ε_{y21}	0	$0.1\mu_{y21}$	Normal
ε_{z1}	0	$0.1\mu_{z1}$	Normal
ε_{z2}	0	$0.1\mu_{z2}$	Normal

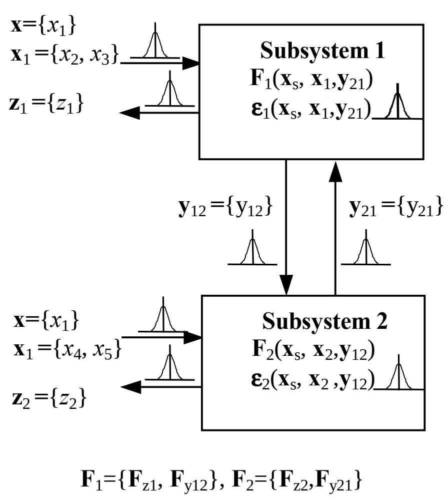

Figure 3 Information Flow – Example 1

For this example, the optimization model without uncertainty is represented as:

Find: the values of $x_1 \sim x_5$

Minimize: z_2

Subject to: $11 - z_1 \leq 0$ and $12 - z_2 \leq 0$

When considering uncertainty, the optimization model is converted to a robust design model as:

Find: the mean values $\mu_{x1} \sim \mu_{x5}$

Minimize: $w_1 \dfrac{\mu_{z2}}{\mu^*_{z2}} + w_2 \dfrac{\sigma_{z2}}{\sigma^*_{z2}}$ (objective)

Subject to: $11 - (\mu_{z1} + k\sigma_{z1}) \leq 0$ (constraint 1)

$12 - (\mu_{z2} + k\sigma_{z2}) \leq 0$ (constraint 2)

where w_1 and w_2 in the above model are the weighting factors with $w_1 + w_2 = 1$. k is chosen to be 1 which indicates that with 84.13% probability, the constraint will be satisfied under the assumption that constraint functions are normally distributed. μ^*_f (obtained by $w_1 = 1$ and $w_2 = 0$) and σ^{*2}_f (obtained by $w_1 = 0$ and $w_2 = 1$) are the ideal solutions used to normalize the two aspects in the objective, i.e., optimizing the mean performance and minimizing performance deviations.

To compare different methods, the robust design results from the proposed hierarchical approach, all-in-one approach with the SUA, all-in-one approach with the CSSUA, and all-in-one approach with the Monte Carlo Simulation (MCS) are listed in Table 2. The MCS is used as the baseline for comparison. 1000 simulations are used for each uncertainty analysis at system level.

Table 2. Robust Design Results from Different methods

Optimum results	Hierarchical approach	All-in-one approach (SUA)	All-in-one approach (CSSUA)	All-in-one approach (MCS)
μ_{x1}	2.1232	1.9999	2.0280	1.9554
μ_{x2}	1.8918	1.6539	1.5076	1.6395
μ_{x3}	0.8368	1.0208	1.0017	1.0243
μ_{x4}	0.0038	0.0	0.0	0.0011
μ_{x5}	3.2643	3.3948	3.5276	3.4269
Objective	4.16296	4.13990	4.17925	4.13498
Constraint 1	-1.1909	-0.2293	-0.0009	0.0
Constraint 2	-0.0190	0.0	-0.0008	0.0
Number of system-level analyses	0	81	0	780000
Number of subsystem 1 analyses	7825	324	626	0
Number of subsystem 2 analyses	9350	324	626	0

It is observed that the optimum solutions obtained by all the testing methods are close to the one from the MCS. In terms of the objective function, besides the MCS that results in the smallest objective value, the best one is the all-in-one approach with the SUA, followed by the hierarchical approach. In terms of computational demand, the hierarchical approach and the all-in-one approach with the CSSUA do not require any system level analysis, while the all-in-one approach with SUA needs 81 system level analysis. Between SUA and CSSUA, the former needs more subsystem level analyses than the latter since suboptimization is involved in the former. For this example, the hierarchical approach requires larger number of subsystem analyses than the all-in-one approach because the number of design variables is increased to match not only the means of linking variables, but also the variances and covariances of linking variables. Hierarachical approach, on the other hand, is the only method that enables the concurrency of optimization and analyses among subsystems. The all-in-one approach with CSSUA may implement uncertainty analysis simultaneously among subsystems, but the robust design optimization is still performed at the system level.

5. Conclusion

In this research, a hierarchical approach to robust multidisciplinary design is proposed, which expands the CO approach to the consideration of uncertainty associated with system input and system models. Three level tasks, system level robust optimization, subsystem level optimization, and subsystem level uncertainty analysis are employed to achieve the concurrency of design optimization and analyses for multidisciplinary robust design. System level optimization minimizes the system objective and maintains the compatibility between subsystems by passing system level design variables, means of sharing variables and means and variances of linking variables to suboptimization of each subsystem and receiving discrepancy of linking variables between those calculated in the suboptimization and those set by system level optimization. Suboptimization minimizes the aforementioned discrepancy and maintains the design feasibility associateed with a specific subsystem. Subsystem level uncertainty analysis receives all the necessary information to evaluate the means and variances of system output and feeds them back to suboptimization and system level optimization. By this way, all the optimization and uncertainty analyses are implemented within subsystems

concurrently.
Based on the discussion, it is noted that, to perform multidisciplinary robust design, we may have at least four alternatives, namely, the hierarchical collaborative approach, the all-in-one approach with the SUA, the all-in-one approach with the CSSUA and the all-in-one approach with the MCS. Since Monte Carlo simulation is a very expensive approach, it is rarely used in MDO unless as a baseline for comparison and verification. Therefore the all-in-one with MCS approach is not applicable for MDO in most cases. If we are interested in distributing the analysis and optimization tasks, we can choose the hierarchical collaborative approach. But it should be noted that in some cases the hierarchical approach may suffer convergence difficulty due to the bi-level nature of the approach and the sensitivity to starting points for suboptimization. For certain problems, the all-in-one approach with the SUA and the all-in-one approach with the CSSUA can be more efficient than the hierarchical approach in terms of the number of subsystems analysis, as the case of our example. However, the efficiency of the hierarchical approach versus the all-in-one approach with SUA or CSSUA is problem dependent, depending on many factors, such as the number of disciplines, number of linking variables and sharing variables, and the degree (strong or weak) of coupling between subsystems. Ref. 7 provides some discussions about the efficiency issue related to uncertainty analysis.

6. Acknowledgement

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