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Analytical metamodel-based global sensitivity analysis and uncertainty propagation for robust design

Ruichen Jin
PTO Engineering & IT Solutions, Ford Motor Company

Wei Chen*
Integrated DEsign Automation Laboratory (IDEAL), Northwestern University

Agus Sudjianto
V-Engine Engineering Subsystem Engineering, Ford Motor Company

ABSTRACT

Metamodeling approach has been widely used due to the high computational cost of using high-fidelity simulations in engineering design. Interpretation of metamodels for the purpose of design, especially design under uncertainty, becomes important. The computational expenses associated with metamodels and the random errors introduced by sample-based methods require the development of analytical methods, such as those for global sensitivity analysis and uncertainty propagation to facilitate a robust design process. In this work, we develop generalized analytical formulations that can provide efficient as well as accurate global sensitivity analysis and uncertainty propagation for a variety of metamodels. The benefits of our proposed techniques are demonstrated through vehicle related robust design applications.

INTRODUCTION

The last decade has seen increasing applications of a wide range of metamodeling techniques in engineering design. With metamodels, a designer is interested in not only predicting a response at a new design setting, but also gaining insight into the relationship between a response and the input variables. While it may be easy to interpret a linear or a quadratic regression model by simply inspecting the regression coefficients, it would be difficult to understand models with sophisticated functional forms, such as MARS, neural network methods, Kriging, etc. In fact, to most of users, these models often appear as "black boxes" even though explicit functional forms are usually available. Sampling-based approaches such as Monte Carlo methods can be used to evaluate the contributions of different factors and their interactions (entitled Sensitivity Analysis). However, when using a set of random sample points, the estimations of multivariate integrals include random errors, which could blur the true results. Therefore the plots of main effects and second-order interaction effects, if obtained by Monte Carlo methods, often involve distracting jitters (Roosen, 1995). For large-dimensional problems, the estimations of a large set of multivariate integrals at different values of the variables in concern further compounds the computational efficiency of Monte Carlo Methods.

A related issue in design under uncertainty is how to utilize the metamodels for uncertainty propagation (UP). Studies show that when Monte Carlo methods are used to estimate the performance mean and variance in metamodel-based robust design optimization, an objective function tends to be noisy and the convergence is not guaranteed (Padmanabhan and Batill, 2000). For those relatively expensive metamodeling techniques such as Kriging, Monte Carlo methods could be inefficient in robust design optimization as a large set of samples are needed to evaluate the performance mean and variance repeatedly.

In the statistics community, a number of variance-based methods have been developed in Sensitivity Analysis (SA) to ascertain how much a model depends on each or some of its input parameters (Chan et al., 1997). Although methods like Sobol (Sobol, 1993) and FAST (Saltelli et al, 1999) are capable of computing the so-called Total Sensitivity Indices (TSI), these methods are developed for general functional relationships and still require a large number of samples in many cases. None of the existing SA and UP methods are developed for

* Corresponding Author, weichen@northwestern.edu. Mechanical Engineering, Northwestern University, Evanston, IL 60208-3111, phone 847-491-7019, fax 847-491-3915.

metamodel-based applications. To improve the efficiency as well as accuracy, there is a need for developing analytical sensitivity analysis and uncertain propagation techniques when using metamodels for design purpose.

ANALYTICAL SA AND UP FOR METAMODELS

In this work, we develop analytical methods that allow efficient as well as accurate evaluations in SA and UP for metamodel-based design applications. Our analytical method for SA utilizes the same concept of variance-based total sensitivity analysis. The commonality between SA and UP is the assessment of response variations that involve multivariate integrals over the variation range of input variables. The common underlying principle for the analytical SA and UP methods developed in this work is to combine the analytical results of univariate integrals to evaluate the multivariate integrals that are associated with multvariate tensor-product basis functions. Since most of the commonly used metamodeling techniques follow the form of multivariate tensor-product basis functions, the generalized formulations derived in this work can be extended for a variety of metamodeling techniques. The detailed descriptions of our techniques are provided in Jin (2003), only the basic ideas and concepts are introduced here.

TOTAL SENSITIVITY INDICES (TSI)

In SA, the *local* sensitivity stands for the *local* variability of the output by varying input variables one at a time near a given central point, which involves partial derivatives. The *global* sensitivity, however, stands for the *global* variability of the output over the entire range of the input variables and hence provides an overall view on the influence of inputs on the output. With variance-based SA, a function is decomposed through functional *analysis of variance* (ANOVA) decomposition (Sobol, 1993; Owen, 1992) into increasing order terms, i.e., first-order terms (main effects) depending on a single variable, higher-order terms (interaction effects) depending on two or more variables, i.e.

$$f(\mathbf{x}) = f_0 + \sum_{i=1}^{M} \phi_i(x_i) + \sum_{i_1=1}^{M} \sum_{i_2 \geq i_1}^{M} \phi_{i_1 i_2}(x_{i_1}, x_{i_2}) + \ldots + \phi_{1 \ldots M}(x_1, \ldots, x_M)$$

Functional ANOVA decomposition provides information on *how* a particular input variable affects the response and *how* the variable interacts with other variables, etc. It should be noted that ANOVA decomposition is based on the assumption that the input variables are statistically independent. In other words, their joint probability density function (PDF) is a product of individual or marginal PDF of all variables, i.e.,

$$p(\mathbf{x}) = p(x_1, x_2, \ldots x_M) = \prod_{i=1}^{M} p_i(x_i) \,,$$

where $p_i(x_i)$ (i=1,2,...,M) is the individual density function for each variable.

Based on the orthogonality feature of the decomposition terms, it can be proved that the variance V of f can be expressed as the summation of the *effect variances* $V_{i_1 \ldots i_s}$ of $\phi_{i_1 \ldots i_s}$ (s^{th}-order effect for $x_{i_1}, \ldots, x_{i_s}$),

$$V = \sum_{s=1}^{m} \sum_{i_1 < \ldots < i_s} V_{i_1 \ldots i_s} = \sum V_i + \sum_{i_1 < i_2} V_{i_1 i_2} + \ldots + V_{1,2,\ldots,m} \,,$$

where,

$$V = Var\{f(\mathbf{X})\} = \int f^2(\mathbf{x}) p(\mathbf{x}) d\mathbf{x} - f_0^2$$

$$V_{i_1 \ldots i_s} = Var\{\phi_{i_1 \ldots i_s}(x_{i_1}, \ldots, x_{i_s})\} = \int \phi^2_{i_1 \ldots i_s}(x_{i_1}, \ldots, x_{i_s}) \prod_{j=i_1}^{i_s} [p_j(x_j) dx_j]$$

Global sensitivity indices are defined to be effect variances normalized by V, i.e., the ratios

$$S_{i_1 \ldots i_s} = V_{i_1 \ldots i_s} / V \,.$$

The first order index (s=1) S_i, called *main sensitivity index* (MSI), is for main effects of variables, and higher-order index (s≥2) $S_{i_1 \ldots i_s}$, called *interaction sensitivity index* (ISI), are for interaction effects among variables. To escape the dimensionality curse without losing important information in interaction effects, Homma and Saltelli (1996) applied the ‘freezing unessential variables’ approach (Sobol, 1993) to investigate the total influence of each individual variable induced both by its main effect and by the interaction effects between it and other variables. This is achieved by partitioning the variables into x_i and its complementary set $\mathbf{x}_{\sim i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_M)$. The *total sensitivity index* (TSI) for variable x_i is given by

$$S_{Ti} = S_i + S_{i,\sim i} = 1 - S_{\sim i} \,,$$

where $S_{i,\sim i}$ is the sum of all the $S_{i_1 \ldots i_s}$ that involve the index i and at least one index from the complementary set (1,...,i-1, i+1,...,M); $S_{\sim i}$ is the sum of all the $S_{i_1 \ldots i_s}$ terms which do not involve the index i.

SUBSET DECOMPOSITION AND UNCERTAIN PROPAGATION

Suppose a performance function can be expressed as $Y = f(\mathbf{x_D}, \mathbf{X_R})$, $\mathbf{x_D}$ is a set of design variables and $\mathbf{X_R}$ is a set of noise variables. Uncertainty propagation is to study the uncertainty in Y caused by the uncertainty in noise variables $\mathbf{X_R}$. In robust design, we are interested in performance mean and performance variance, the two uncertainty quantities can be defined as,

$$\mu_Y(\mathbf{x_D}) = E[f(\mathbf{x_D}, \mathbf{X_R})] = \int f(\mathbf{x}) p_{\mathbf{R}}(\mathbf{x_R}) d\mathbf{x_R} \ ,$$

$$\sigma_Y^2(\mathbf{x_D}) = Var[f(\mathbf{x_D}, \mathbf{X_R})] = \int [f(\mathbf{x}) - \mu_Y(\mathbf{x_D})]^2 p_{\mathbf{R}}(\mathbf{x_R}) d\mathbf{x_R} \ ,$$

where $p_{\mathbf{R}}(\mathbf{x_R})$ are the joint PDF for the noise variables. Based on subset decomposition, the total variance of the output can be decomposed into,

$$V = \widehat{V}_{\mathbf{D}} + \widehat{V}_{\mathbf{R}} + \widehat{V}_{\mathbf{DR}} \ ,$$

where $\widehat{V}_{\mathbf{D}}$ is the subset variance due to design variables, $\widehat{V}_{\mathbf{R}}$ is the subset variance due to noise variables, $\widehat{V}_{\mathbf{DR}}$ is the subset variance due to the interaction between design variables and noise variables. The subset decomposition provides a genetic form for ANOVA decomposition and it is the basis of uncertainty propagation.

SUMMARY OF ANALYTICAL FUNCTIONS TO EVALUATE

Based on the above descriptions, we can summarize the important functions to evaluate for both global SA and UP as the following:

$$f_0 = \int f(\mathbf{x}) \prod_{i=1}^{M} [p_i(x_i) dx_i] \ ,$$

$$\widehat{f}_{\mathbf{U}}(\mathbf{x_U}) = \int f(\mathbf{x}) \prod_{i \notin \mathbf{U}} [p_i(x_i) dx_i] \ ,$$

$$\widehat{V}_{\mathbf{U}} = \int \widehat{f}_{\mathbf{U}}^{\,2}(\mathbf{x_U}) \prod_{i \in \mathbf{U}} [p_i(x_i) dx_i] - f_0^{\,2} \ ,$$

$$\mu_Y(\mathbf{x_D}) = \int f(\mathbf{x}) \prod_{i \in \mathbf{R}} [p_i(x_i) dx_i] \ ,$$

$$\sigma_Y^2(\mathbf{x_D}) = \int [f(\mathbf{x}) - \mu_Y(\mathbf{x_D})]^2 \prod_{i \in \mathbf{R}} [p_i(x_i) dx_i] \ ,$$

where **U** denotes the index set of variables in $\mathbf{x_U}$; **D** denotes the index set of the design variables in $\mathbf{x_D}$; **R** denotes the index set of the noise variables in $\mathbf{x_R}$. The commonality between SA and UP is the assessment of response variations that involve multivariate integrals over the variation range of input variables.

MULTIVARIATE TENSOR-PRODUCT BASIS FUNCTIONS

Define a set of multivariate tensor-product basis functions $B_i(\mathbf{x})$ to be the product of *M* univariate basis function $h_{il}(x_l)$, i.e.,

$$B_i(\mathbf{x}) = \prod_{l=1}^{M} h_{il}(x_l), \ i = 1, 2, ..., N_b$$

.

Note here $h_{il}(x_l)$ could be equal to 1. Then a special category of functions can be defined as a linear combination of these multivariate tensor-product basis functions, i.e.,

$$f(\mathbf{x}) = a_0 + \sum_{i=1}^{N_b} a_i B_i(\mathbf{x}) = a_0 + \sum_{i=1}^{N_b} [a_i \prod_{l=1}^{M} h_{il}(x_l)] \ ,$$

where a_i (i =0, 1,..., N_b) are constant coefficients. Most of the widely used metamodeling techniques, including polynomial regression model, MARS, Radial Basis Function (with Gaussian basis functions), Kriging, and any combinations of these models, belong to this category. The advantage of generalizing the forms of metamodels to the multivariate tensor-product basis functions is that the multivariate integrations involved in global SA and UP can be analytically reduced to a set of univariate integration, which can be either evaluated analytically or numerically with high accuracy.

GENERALIZED ANALYTICAL SA AND UP

It can be shown that the mean of $f(\mathbf{x})$ and the subset main effect of variables $\mathbf{x_U}$ can be obtained, respectively,

$$f_0 = a_0 + \sum_{i=1}^{N_b} (a_i \prod_{l=1}^{M} C_{1,il}) \ ,$$

$$f_{\mathbf{U}}(\mathbf{x_U}) = a_0 + \sum_{i=1}^{N_b} [a_i \prod_{l \notin \mathbf{U}} C_{1,il} \prod_{l \in \mathbf{U}} h_{il}(x_l)] \ ,$$

where, $C_{1,il} = \int h_{il}(x_l) p_l(x_l) dx_l$; and $\mathbf{U} = (k_1, k_2, ..., k_q)$ denotes the indices of $\mathbf{x_U}$.

Furthermore, the variance of $f(\mathbf{x})$ and the subset variance corresponding to $\mathbf{x_U}$ are derived as, respectively,

$$V = \sum_{i_1=1}^{N_b} \sum_{i_2=1}^{N_b} \left\{ a_{i_1} a_{i_2} \prod_{l=1}^{M} (C_{1,i_1 l} C_{1,i_2 l}) \{ \prod_{l=1}^{M} [C_{2,i_1 i_2 l} / (C_{1,i_1 l} C_{1,i_2 l})] - 1 \} \right\} ,$$

$$V_{\mathbf{U}} = \sum_{i_1=1}^{N_b} \sum_{i_2=1}^{N_b} \left\{ a_{i_1} a_{i_2} \prod_{l=1}^{M} (C_{1,i_1 l} C_{1,i_2 l}) \{ \prod_{l \in \mathbf{U}} [C_{2,i_1 i_2 l} / (C_{1,i_1 l} C_{1,i_2 l})] - 1 \} \right\} ,$$

where, $C_{2,i_1 i_2 l} = \int h_{i_1 l}(x_l) h_{i_2 l}(x_l) p_l(x_l) dx_l$.

Using these two equations, all the variances and thereby all the sensitivity indices, can be obtained,

For robust design, the performance mean and variance can be evaluated, respectively by,

$$\mu_Y(\mathbf{x_D}) = a_0 + \sum_{i=1}^{N_b}[a_i \prod_{l \notin \mathbf{D}} C_{1,il} \prod_{l \in \mathbf{D}} h_{il}(x_l)],$$

$$\sigma_Y^2(\mathbf{x_D}) =$$
$$\sum_{i_1=1}^{N_b}\sum_{i_2=1}^{N_b}\left\{a_{i_1}a_{i_2}\prod_{l \notin \mathbf{D}}(C_{1,i_1 l}C_{1,i_2 l})\{\prod_{l \notin \mathbf{D}}[C_{2,i_1 i_2 l}/(C_{1,i_1 l}C_{1,i_2 l})]-1\}\prod_{l \in \mathbf{D}}[h_{i_1 l}(x_l)h_{i_2 l}(x_l)]\right\}$$

ROBUST DESIGN OF ENGINE PISTON

PROBLEM DEFINITION

The Noise, Vibration and Harshness (NVH) characteristic of the vehicle engine is one of the critical elements of customer dissatisfaction. Piston slap is an unwanted engine noise that is the result of piston secondary motion, that is, the departure of the piston from the primary motion prescribed by kinematic analysis. By predicting the secondary motion of the piston, the power cylinder system can be modified early in the engine design phase, when more design options exist, to prevent this noise. However, the piston design changes made to improve piston guidance in the cylinder and, thereby, reduce piston slap noise often have a negative impact on the friction between piston and cylinder, causing additional power loss. Furthermore, some uncontrollable factors such as piston-bore clearance, location of combustion peak pressure, etc. can have significant effects on slap noise and piston friction. Therefore, it is desirable to find a design that not only makes a good tradeoff between slap noise and piston friction but also is insensitive to the variation of these system noises. Hoffman, et. al. (2002) presented a comprehensive simulation model for piston secondary motion analysis and used it in the study of the design robustness of engine piston via First Order Reliability Method (FORM). Li and Sudjianto (2003) developed penalized likelihood Kriging model and applied their new method to create a surrogate model to replace the piston simulation model for design purpose. In this study, our analytical global SA and UP techniques are applied to the same piston design while using metamodels for robust design.

Two performance responses, i.e., the sound power level of piston slap noise (*slap noise* y_n for simplicity) and the power loss due to piston friction (*piston friction* y_f for simplicity), are considered. The changes in piston design are made to improve piston guidance in the cylinder and, thereby, to reduce slap noise that often has a negative impact on the piston friction. Four piston geometric parameters, namely, skirt length (SL), skirt profile (SP), skirt ovality (SO) and pin offset (PO) are considered as design variables. The ranges of the design variables are provided in Table 1.

Table 1. Design Variables in Engine Piston Design

Variable	Description	Nominal Value	Lower Bound	Upper Bound	Unit
SL	Skirt Length	23.07	21	25	millimeter
SP*	Skirt Profile	3	1	3	/
SO*	Skirt Ovality	2	1	3	/
PO	Pin Offset	0.9	0.5	1.3	millimeter

In addition, two noise factors are considered in this work, i.e., the piston-to-bore clearance (CL) and the location of combustion peak pressure (LP). The clearance changes due to temperature (which varies as the engine warms), cylinder bore distortion, piston wear, and the collapse of piston skirt; the location changes due to spark timing, which can vary from cycle-to-cycle, and bank-to-bank. As shown in Table 2, both the clearance (CL) and the location (LP) are assumed to follow normal distributions.

Table 2. Noise Variables in Engine Piston Design

Variable	Descript.	Distribution	Mean	STD	Unit
CL	Piston-to-bore Clearance	Normal	50	11	micrometer
LP	Location of Combustion Peak Pressure	Normal	14.5	1	degree

DEVELOPMENT OF METAMODELS

A metamodel is a cheap-to-compute approximation to the computationally expensive multi-body dynamic model constructed using the results of computer experiments. The metamodel is needed because robust design requires many runs for which direct computation using the multi-body dynamic model like ADAMS is computationally prohibitive.

The sequential metamodeling approach is applied to create metamodels for two performance functions, i.e., slap noise y_n and piston friction y_f. Here we use the Kriging method as the metamodeling technique. In the first stage, a 30×6 (30 runs for 6 variables) optimal LHD based on the ϕ_p criterion (p=2, t=1) is generated by the optimal DOE algorithm (Jin et al. 2003); at each sequential stage, 30 new sample points are generated by the sequential sampling method. The accuracy (measured by two types of metrics) of the metamodels created in each stage is shown in the following two tables.

Table 3. The Accuracy of Metamodels for Slap Noise

	R^2	*RAAE*	R_{CV}^2	$RAAE_{CV}$
First Stage	0.9846	0.1011	0.9234	0.2055
Second Stage	0.9987	0.0292	0.9987	0.0304
Third Stage	/	/	0.9997	0.0134

Table 4. The Accuracy of Metamodels for Piston Friction

	R^2	*RAAE*	R_{CV}^2	$RAAE_{CV}$
First Stage	0.999964	0.00442	0.999951	0.00508
Second Stage	/	/	0.999996	0.00116

From the tables, it is noted that with 3 stages, the metamodel for slap noise is highly accurate; and with 2 stages, the metamodel for piston friction is extremely accurate.

INTERPRETATION OF THE METAMODELS

Once the metamodels are created, our developed analytical global sensitivity analysis techniques are used to provide insights into the relation between the design variables, the noise variables, and the responses. The sensitivity indices of each variable are shown in Figs. 1 and 2, respectively for the two responses. In the figures, 'main' stands for the sensitivity indices for main effect of each variable; 'total' stands for the total sensitivity indices (TSI) for each variable; 'D' stands for design variables and 'N' stands for noise variables. The sequence of the variables is arranged based on the relative importance in terms of TSI. From the figures, we can find that the noise variables clearance (CL) has significant influence on the variability of both the slap noise and the piston friction. From Fig. 1, it can be found that the total sensitivity indices are considerably larger than the sensitivity indices of main effects, which means that the interaction between variables are significant.

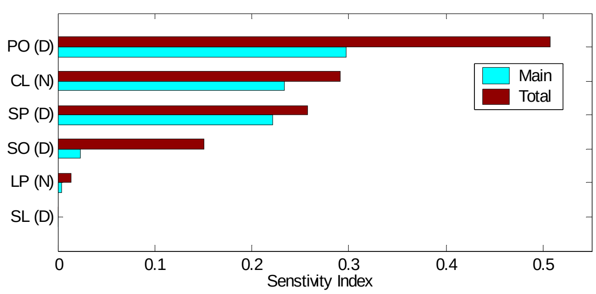

Figure 1. The Sensitivity Indices for Slap Noise

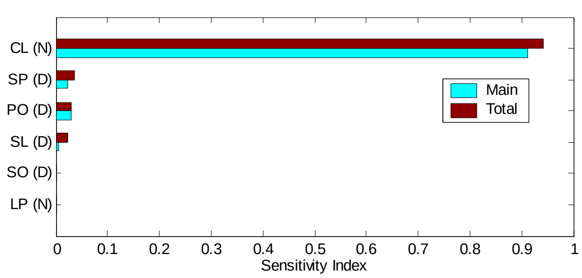

Figure 2. The Sensitivity Indices for Piston Friction

Figs. 3 and 4 further illustrate the contribution (to the variability of given responses) of the *subset* main effects of design variables (including all the main effects of the design variables and all the interactions between them), the *subset* main effects of design variables (including all the main effects of the noise variables and all the interactions between them), and the *subset* interaction between noise variables and design variables (including all the interactions that involve at least one of the noise variables and one of the design variables). It is found that interactions between design variables and noise variables are non-negligible. This is a desired feature in robust design. By properly choosing the values of design variables, these interactions could dampen the variability caused by the noise variables.

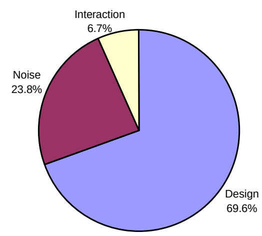

Figure 3. Contribution of Noise Variables, Design Variables, and Interaction to Slap Noise

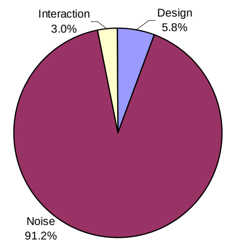

Figure 4. Contribution of Noise Variables, Design Variables, and Interaction to Piston Friction

Figs. 5 and 6 show respectively the main effect of each variable on slap noise and piston friction (all the variables are normalized). Main effects of the noise variable clearance (CL) on both responses are very significant and nonlinear. It shows that the order of nonlinearity is higher than two(2). Due to this nonlinearity, using the first-order Taylor expansion method for evaluating the performance variation under uncertainty most likely will not be accurate enough. Furthermore, from main effects of design variables, in particular skirt profile (SP) and pin offset (PO), we find that the trends for minimum slap noise differ from those for minimum piston friction, thus complicating optimization for both noise and friction. A trade-off is therefore needed.

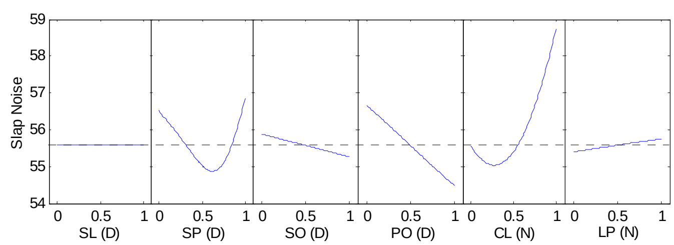

Figure 5. Main Effects of Variables on Slap Noise

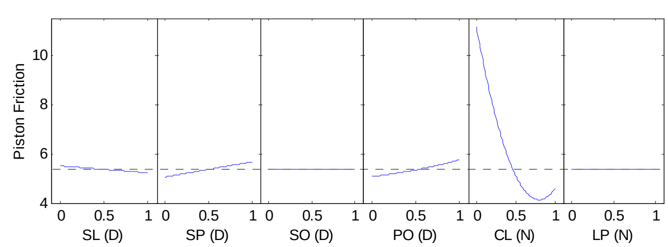

Figure 6. Main Effects of Variables on Piston Friction

Figs. 7 and 8 show respectively the interaction between clearance (CL) and pin offset (PO) for slap noise and the interaction between clearance (CL) and skirt length (SL) for piston friction. Based on the robust design concept, these interactions between design variables and noise variables are desirable as they may lead to a reduction in variability of slap noise and piston friction incurred by noise variables. For instance, choosing the upper bound of pin offset (PO) and the low bound of skirt length (SL) will lead to smaller variability of slap noise and piston friction, respectively.

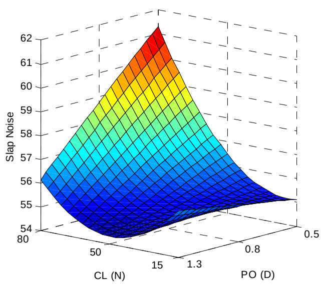

Figure 7. Interaction Between CL and PO for Slap Noise

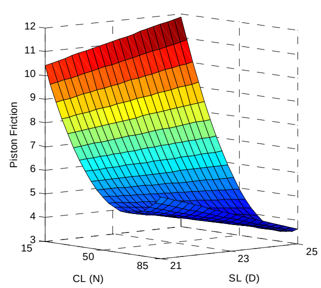

Figure 8. Interaction Between CL and SL for Piston Friction

ROBUST DESIGN OPTIMIZATION

The robust design of engine piston is then modeled using the robust design formulation as shown in Fig. 9.

Given
Distribution of noise factors**:** CL, LP
Metamodels for y_n and y_f
Find
Piston Geometric Parameters: SL, SP, SO, PO
Satisfy
Boundaries of the variables
Minimize
Robust Design Metrics
$\{F_n=\mu_{yn}+3\sigma_{yn}, F_f=\mu_{yf}+3\sigma_{yf}\}$
y_n – slap noise; y_f – piston friction.

Figure 9. Robust Design Formulation for Engine Piston

In this case study, to reduce the means of slap noise and piston friction, as well as their variations, the following robust design metrics for slap noise and piston friction are chosen to be minimized:

$$F_n = \mu_{y_n} + 3\sigma_{y_n} \text{ and } F_f = \mu_{y_f} + 3\sigma_{y_f},$$

If slap noise and piston friction are normally distributed, F_n (3-Sigma Noise) and F_f (3-Sigma Friction) represent the upper 99.865 percentiles of the responses. In this way, the four objectives, including the means (μ_{y_n} and μ_{y_f}) of slap noise and piston friction, and their variances (σ_{y_n} and σ_{y_f}), have been reduced to two objectives (F_n and F_f). The analytical uncertainty propagation method is applied to evaluate the means (μ_{y_n} and μ_{y_f}) and variances (σ_{y_n} and σ_{y_f}).

The Pareto Frontier of the two objectives, i.e., F_n and F_f, shown in Figure 10, is constructed by simply minimizing F_n and expressing F_f in the form of inequality constraint, i.e., for any constant ε,

Minimize F_n

Subject to: $F_f \leq \varepsilon$

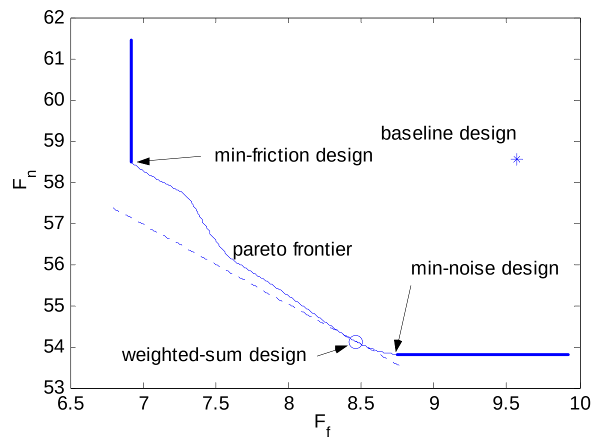

Figure 10. Pareto Frontier for Slap Noise VS Piston Friction Tradeoff Analysis

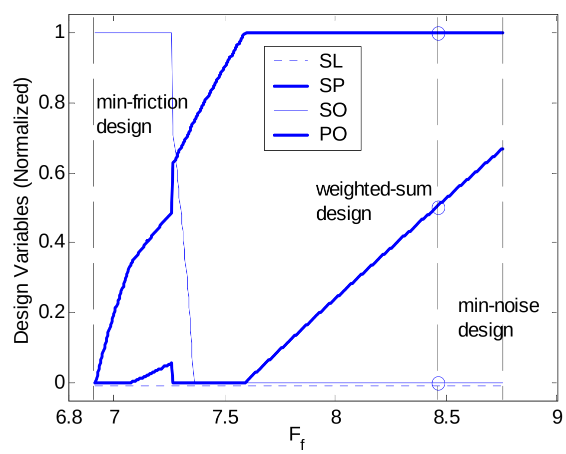

Figure 11. The Values of Design Variables In Pareto Frontier

The solution of design variables corresponding to each point on the pareto frontier in Fig. 10 is shown in Fig. 11. It is noted that while the values of other design variables change in accordance with the value of F_f (or the point on the pareto frontier), the value of SL (skirt length) is kept at its lower bound. The reason for this is that while SL does not have any influence on slap noise (see Figs. 3 and 5), the lower bound of SL will lead to less variability of piston friction, i.e., σ_{y_f} and thus smaller value of F_f (see Fig. 10, also note SL could has a small impact on μ_{y_f}, see Fig. 6). Given the designer's preference on how to make tradeoff between the two different performance characteristics, an optimum robust design can be chosen. For instance, if we consider three different preferences: 1). Minimizing Slap Noise (F_n), 2). Minimizing Friction (F_f), and 3). Weighted-sum trade-off between F_n and F_f. The first two are solved using optimization with a single objective (either F_n or F_f). The third preference is captured by a weighted sum of two objectives, e.g.,

$$\text{Minimize: } w_1 \frac{F_n}{F_{n\min}} + w_2 \frac{F_f}{F_{f\min}},$$

where w_1 and w_2 are the weights, F_{nmin} and F_{fmin} are the minimum values of F_n and F_f, i.e., the results of preference 1 and 2. Table 10 shows the solutions based on the three preferences. Here for weighted-sum preference, w_1 is chosen to be 0.8 and w_2 to be 0.2, i.e., we put more preference on slap noise. It is noted that there are multiple solutions for preferences 1 and 2. In fact all the points in the horizontal dash-dot line (F_f ranging from 8.7565 to 9.9887) and all the points in the vertical dash-dot line (F_n ranging from 58.4989 to 61.4640) are solutions for preference 2. This happens because SL does not have any influence on slap noise while SO does not have any influence on piston friction, which means changing one of the two variables (with other variables fixed) will keep the value of one response while changing the other response. The result shown in Table 5 for preferences 1 and 2 are at the right and left end of the Pareto frontier (solid line), respectively. The left end point of the Pareto frontier is the solution based on preference 2 and the right end point of the Pareto frontier is the solution based on preference 1. It should also be noticed that the weighted-sum approach does not necessarily reflect the design preference correctly (see, e.g., Scott, 2000; Chen, 1999). One problem for weighted sum strategy is that if the Pareto frontier is nonconvex, as is the case in this example (see Fig. 10), weighted-sum strategy will miss some design solutions in the Pareto frontier that is not convex.

Table 5. Solution of Robust Design

		Baseline Design	Min-Noise Design	Min-Friction Design	Weighted-Sum Design
Design Variables	**SL**	23.07	21.00	21.00	21.00
	SP	3.000	2.339	1.000	2.008
	SO	2.000	1.000	1.775	1.000
	PO	0.900	1.300	0.500	1.300
Robust Design Metrics	μ_{yn}	56.8525	52.7253	57.2628	53.0419
	σ_{yn}	0.5654	0.3634	1.0143	0.3629
	F_n	58.5488	53.8154	60.3056	54.1305
	μ_{yf}	5.6468	5.9983	4.9176	5.8978
	σ_{yf}	1.3056	0.9193	0.6690	0.8546
	F_f	9.5635	8.7563	6.9245	8.4616

While analytical uncertainty propagation method is the most accurate method to evaluate mean and variance, Monte Carlo method can provide estimations of these uncertainty characteristics. However, the estimation from Monte Carlo method will include noise, which would cause convergence difficulty for optimization tools. Figs 12a and 13a show the results of mean and variance of slap noise with the change PO and SO, which are evaluated by our proposed analytical approach. Figs 12b and 13b show the evaluations by Monte Carlo method (with 1000 random sample points). In all these figures, SL and SP, two less important variables, are fixed to 23 and 2, respectively. Compared to the results from the analytical method, the results from Monte Carlo Method, especially STD, have a considerable noise. We test the use of Monte Carlo method for the same robust design optimizations. We found even with multiple starting points, in most cases, the optimizer fails to converge. Furthermore, analytical method is more efficient than Monte Carlo method. For instance, to evaluate the 21×21 grid points in the figures, Monte Carlo method takes 32 seconds (PIII 650MHZ, MATLAB), while analytical method takes merely 1.7 seconds.

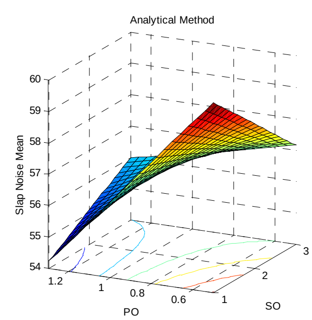

Figure 12b. Mean of Slap Noise Evaluated by Analytical Method

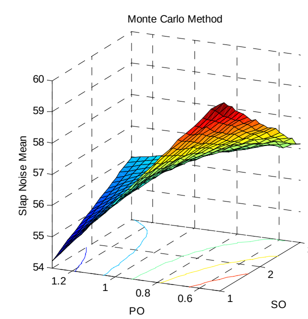

Figure 12b. Mean of Slap Noise Evaluated by Monte Carlo Method

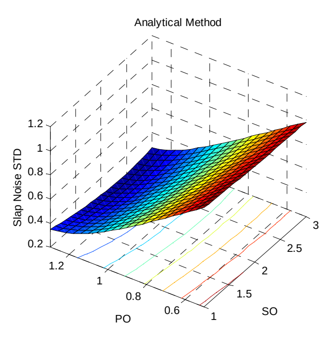

Figure 13a. STD of Slap Noise Evaluated by Analytical Method

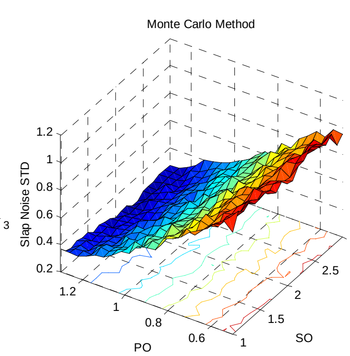

Figure 13b. STD of Slap Noise Evaluated by Monte Carlo Method

CLOSURE

In this work, a robust design procedure that integrates optimal DOE, sequential metamodeling, analytical global sensitivity analysis, and analytical uncertainty propagation, is applied to vehicle design problems. It is illustrated through the example problem that this approach is very useful for designing systems that demands significant computational resources for system simulation. The first major advantage of our approach is the improvement of computational efficiency by using metamodels instead of the actual sophisticated simulation programs. By using this approach, the computational time required for optimization and the evaluation of the uncertainty characteristics of performance responses in robust design can be reduced significantly. If the simulation program is used directly, robust design optimization is a double loop process, i.e., the outer loop for optimization and the inner loop for the evaluation of uncertainty characteristics. In such case, the total number of simulations would be huge, making robust design optimization computationally prohibitive. With the response surface modeling approach, we only use 90 runs for the piston design problem (the time spent in fitting metamodels and the optimization process is relatively small). This significant reduction of computational demand makes the robust design optimization of complex systems more tractable. As illustrated in the piston design case study in which different design preferences are considered, metamodels can be reused for design evaluations or optimizations with changed criteria. Therefore, the approach facilitates the exploration of design solutions through exercising different design scenarios. Ultimately, it helps to improve the design productivity and to shorten the time-to-market.

The fundamental contribution of this work is the development of analytical techniques for assessing the global sensitivity and performance distribution characteristics for a wide variety of metamodels. Our approach provides more accurate as well as more efficient evaluation techniques. The knowledge obtained though global sensitivity analysis provides useful guidance in robust design. Analytical uncertainty propagation reduces the noises associated with sampling methods and greatly facilitates the convergence of robust design optimization.

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