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A VARIATION-BASED METHOD FOR PRODUCT FAMILY DESIGN

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Product family design entails all of the challenges of product design while adding the complexity of coordinating the design of multiple products in an effort to maximize commonality across a set of products without compromising their individual performance. This paper presents the Variation-Based Platform Design Method (VBPDM) for product family design, which aims to satisfy a range of performance requirements using the smallest variation of the product designs in the family. In the first stage of the VBPDM, the product platform around which the product family is to be developed is identified. The product platform is common to all of the products in the family and represents the maximum standardization possible considering the variety of performance requirements that must be satisfied. To satisfy the range of performance requirements for the product family, a ranged set of solutions is found using variation-based modeling. A compromise Decision Support Problem (DSP) is formulated to solve the tradeoff between satisfying the variety requirement and maximizing platform commonality. Platform commonality is achieved by introducing a commonality goal that seeks to minimize the deviation of the input design variables while satisfying the range of performance requirements. Those design variables that show small deviations are held constant to form the product platform. In the second stage of the VBPDM, each individual product is designed around the common platform such that the functional requirements for each product in the family are best satisfied. As an example, the proposed method is used to develop a family of universal electric motors designed to meet a range of torque requirements. The results are compared against previous work on the same example.

Keywords: Product family design; Product platform; Commonality; Universal electric motor

NOMENCLATURE

DSP	Decision Support Problem
VBPDM	Variation-Based Platform Design Methodology
PPCEM	Product Platform Concept Exploration Method
p	Number of products in the product family ($j = 1, \ldots, p$)
n	Number of design variables for each product ($k = 1, \ldots, n$)
μ_{x_k}	Mean value of the kth design variable in the product family
σ_{x_k}	Standard deviation of the kth design variable in the product family

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μ_{b_i}	Mean value of the *i*th desired performance in the product family
σ_{b_i}	Standard deviation of the *i*th desired performance in the product family
μ_{h_i}	Mean value of the *i*th actual performance in the family
σ_{h_i}	Standard deviation of the ith actual performance in the family
$\mathbf{X}$	Vector of all the design variables for a product $(x_1, \ldots, x_n)$
$h_{ij}(\mathbf{X})$	The *i*th performance of the *j*th product
b_{ij}	The *i*th performance requirement of the *j*th product
W_t	Weights for the different objectives in the compromise DSP

1 INTRODUCTION

In this age of a "buyer's market" in which manufacturing firms must satisfy individual customer requirements, many of the current management strategies address developing single products as rapidly as possible. Meyer and Lehnerd [12] note that the focus on individual customers and products often results in "a failure to embrace commonality, compatibility, standardization, or modularization among different products or product lines." The end result is a proliferation of product variety, increasing costs and reducing profit margins. Meyer and Utterback [13] argue for a broader approach to managing new products. According to them, concentrating the design efforts at the level of the product family, and on the development and sharing of key components and assets within a family, is of vital importance. This approach is in keeping with the evolving paradigm of mass customization (see, *e.g.* Pine [17]; Kotha [8]).

Meyer and Utterback [13] give the following definitions of *Product Platform* and *Product Family.* A *product platform* encompasses the design and components shared by a set of products. An effective platform is the core of a successful product family, serving as the foundation for a series of closely related products. Products that share a common product platform but have specific features and functionality to satisfy different sets of customers form a *product family.* A product family typically addresses a market segment, while specific products within the family target niches within that segment.

It becomes clear from the above definitions that a product family should be designed to satisfy a range of functional requirements, and the ideal product platform should include all of the non-differentiating features of the products in the family. In other words designers should attempt to standardize all those components that need not be essentially varied to satisfy the varying functional requirements of the products in the family (*cf.* Kota and Sethuraman [7]). For example, Sanderson and Uzumeri [20] discuss how Sony has managed to effectively design and develop three Walkman® platforms to maintain dominance of the portable audio stereo market. Selection of a good platform the first time is very important since it provides the foundation for the product family and changing a platform can involve significant capital expenditure, as companies have to redesign their existing products (*cf.* Meyer and Lehnerd [12]).

More and more engineering tools and methods are being developed for designing product families. In the modular design approach to product family design, standardization of component parts and sub-assemblies across different products in the product family is attempted by focusing on developing modular product architecture. Ulrich and Eppinger [28] define an integral architecture to have a complex mapping between functions and components, whereas a modular architecture exhibits one-to-one mapping between functions and components. Along the same lines, Pine [17] suggests that the best method to achieve maximum individual customization while minimizing cost is through creation of modular components that can be configured into a wide variety of end products and services. Stone, *et al.* [27] propose a heuristic method to identify modules from a functional description of a product. McAdams, *et al.* [11] introduce a method that uses customer needs across a range of products, translated into functional requirements, to explore design solutions at a functional level. Such an approach allows design synthesis of modules, which are independent of and prior to the existence of form and structure of a particular product. Allen and Carlson-Skalak [1] develop a methodology for designing modular products, which involves identification of modules of a previous generation of product. While most of the product family design concepts are applicable to products that are modular with respect to functions (*cf.* Siddique, *et al.* [21]), Kota and Sethuraman [7] note that generally, a modular design tends to have more components than a multi-functional design performing the same overall function. An "optimum" balance between the functional content that each part should have and the level of commonality that should exist within a product family is an important research topic that needs to be addressed.

The other approach to designing product families is to develop a parametrically scalable product platform (Rothwell and Gardiner [18, 19]). This platform can then be scaled in one or more parameters to develop the product family. Simpson, *et al.* [23] developed the Product Platform Concept Exploration Method (PPCEM) based on this approach. In the PPCEM, the market segmentation grid is employed to help identify suitable scale factors around which the common product platform is "scaled" or "stretched" to satisfy a range of performance requirements. Robust design principles are used to facilitate the design of the common platform by minimizing the sensitivity of the performance to variations in the scale factor(s). The variations of the scale factor(s) reflect the amount of product variety within the product family. Based on a similar principle to that of the PPCEM, the Product Variety Tradeoff Evaluation Method (PVTEM) is presented by Conner, *et al.* [4] to assess appropriate product family tradeoffs using the commonality and performance indices developed in Simpson [25]. In each of the above parametric modeling approaches, the set of the scale factor(s) and that of the common platform parameters are pre-selected, and the selection is independent from the commonality and performance tradeoff evaluation model.

The objective of this paper is to *integrate the selection of the common (platform) parameters and the non-platform variables (scale factors) as a part of the commonality and performance tradeoff procedure in the context of developing parametrically scalable product platforms.* In particular, the following question is asked:

> Given the performance desired from individual products of a family, how can the common product platform and the individual products in the family be designed to maximize the commonality within the product family as well as to best satisfy the design requirements of each product?

The Variation-Based Platform Design Method (VBPDM) introduced in the next section extends the PPCEM to provide a more flexible formulation for modeling a product family. The VBPDM uses variation-based modeling to accommodate flexible design specifications (*i.e.* a range of solutions) to integrate the selection of common (platform) parameters and the scale factors as part of the commonality and performance tradeoff.

2 VARIATION-BASED PLATFORM DESIGN METHOD (VBPDM)

The Variation-Based Platform Design Method (VBPDM) presented in this paper provides a logical extension of the PPCEM to develop a product family represented by a range of performance targets. Given the description of the design variables and their relationship with the resulting performances, *the VBPDM can be used to identify those design variables (scale* *factors) that need to change from product to product in the family to satisfy the changing performance requirements, and those design variables (common platform parameters) that can be held constant for all the products of the family.* Thus the proposed approach first identifies the common product platform, which is designed to maximize the standardization of components across the product family, taking into account the overall range of functional requirements that must be satisfied. The individual products are then subsequently designed around this common platform to satisfy their functional requirements. This results in a range of products that satisfies a range of requirements with maximum commonality.

Figure 1 shows a schematic representation of the VBPDM. The first stage of the VBPDM is the selection of the common product platform from among the set of n design variables for each of the p products in the family. In the second stage, each of the p individual products is designed around the common platform. The details of these two stages are elaborated in the following two sub-sections.

2.1 Stage 1: Platform Selection in VBPDM

In the platform selection stage, a *single* compromise DSP (see Figure 2) is formulated to determine (a) which of the n design variables should be selected as the common platform variables and (b) the optimal values for these variables. The compromise DSP is a hybrid formulation based on mathematical programming and goal programming for solving multi-objective optimization problems (Mistree *et al.* [14]). It is used to determine the values of the design variables that satisfy a set of constraints and achieve, as closely as possible, a set of potentially conflicting goals. In the compromise DSP, goals are formulated as equalities, setting appropriate targets, to evaluate the tradeoffs between multiple goals effectively.

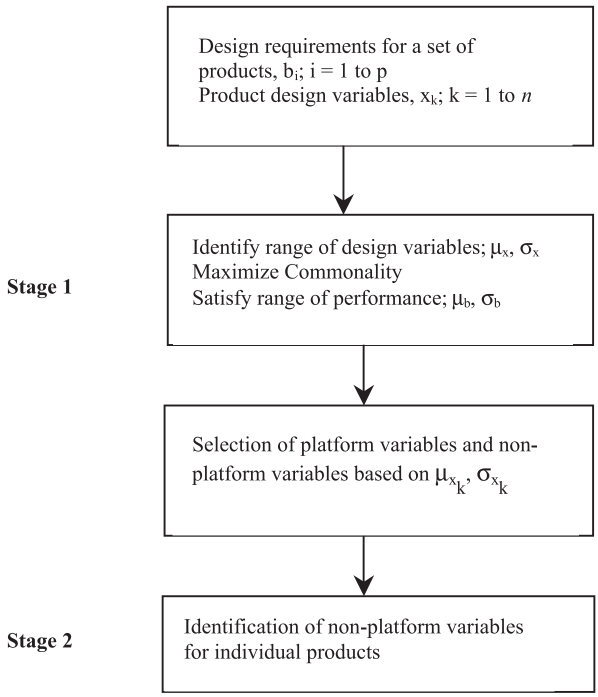

FIGURE 1 Schematic representation of the VBPDM.

Given

Mathematical relationships, and constants

Find

μ_{x_k} k = 1, ..., n

σ_{x_k} k = 1, ..., n *n* is the number of design variables for each of the *j* = 1, ..., *p* products in the product family

Satisfy

The *system constraints* that must be satisfied can be classified as follows:

Equality constraints on performance with a different desired value for each product of the family $h_{ij}(\mathbf{X}) = b_{ij}$

$$\text{This constraint is modeled as } \mu_{h_i} = \mu_{b_i} \text{ and } \sigma_{h_i} = \sigma_{b_i} \quad (2.1)$$

Equality constraints on performance with the same desired value for each product of the family $h_{ij}(\mathbf{X}) = b_i$

$$\text{This constraint is modeled as } \mu_{h_i} = b_i \text{ and } \sigma_{h_i} = 0 \quad (2.2)$$

Inequality constraints on performance with a different limiting value for each product of the family $h_{ij}(\mathbf{X}) \leq b_{ij}$

$$\text{This constraint is modeled as } \mu_{h_i} \leqslant \mu_{b_i} \quad (2.3)$$

Inequality constraints on performance with the same limiting value for each product of the family $h_{ij}(\mathbf{X}) \leq b_i$

$$\text{This constraint is modeled as } h_i(\mathbf{X})_{worst\text{-}case} \leqslant b_i \quad (2.4)$$

The *system goals* that must achieve a specified target as far as possible:

Goal targets for performance, with a different value for each product of the family $h_{ij}(\mathbf{X})/T_{ij} + d_{ij}^{-} - d_{ij}^{+} = 1$

$$\text{This goal is modeled as } \mu_{h_i}/\mu_{T_i} + d_i^- - d_i^+ = 1 \quad (2.5)$$

Goal targets for performance, with the same target value for each product of the family $h_{ij}(\mathbf{X})/T_i + d_{ij}^{-} - d_{ij}^{+} = 1$

$$\text{This goal is modeled as } \mu_{h_i}/T_i + d_i^- - d_i^+ = 1 \quad (2.6)$$

The commonality goal for minimizing the deviation of the system variables, and thus helping in standardization

$$(\sigma_{x_1}/a_1 + \cdots + \sigma_{x_k}/a_n)/n + d^- - d^+ = 0 \quad (2.7)$$

where $a_1, \ldots, a_k$ are the normalizing factors

Minimize

$$Z = \sum_{t=1}^{i+1} W_t(d_t^- + d_t^+);\ \Sigma W_t = 1;\ W_t \geqslant 0$$

FIGURE 2 Compromise DSP for determining the product platform.

To consider all of the products in this single compromise DSP, the variation among the family of products is captured by the deviations of the design variables and the resulting performance deviations using variation-based modeling. The use of variation-based modeling has been reported earlier for robust design applications in Chen *et al.* [3]. For ease of computation, a probabilistic representation is adopted. The design variables of each product are modeled to follow uniform probability distributions represented by two variables: the mean, μ, and the standard deviation, σ. The resulting variation of each performance parameter is desired to match with the desired ranges of performances across the entire product family.

The compromise DSP shown in Figure 2 is formulated to find the means and standard deviations of the design variables that result in a desired range of performance. Constraints and goal targets are imposed on the mean and standard deviation of the performance so as to satisfy the range of performance requirements for the entire product family. On the other hand, the need for commonality requires the use of a minimum set of design variables (non-platform variables) whose deviations help satisfy the range of requirements. Hence, one objective in the compromise DSP is to find the smallest set of design variables whose variation will satisfy the range of performance requirements as best as possible. This is accomplished by creating a goal of minimizing the total deviations in as many design variables as possible (Eq. (2.6)). This goal is called the *commonality goal.*

Because there is a tradeoff between achieving commonality within a product family and satisfying the functional requirements of each product, an Archimedean formulation is used in the compromise DSP in which weights are assigned to the different goals to make the tradeoffs. *Here we assume there is no correlation among the different performance responses.* After solving the compromise DSP, if the standard deviation of a design variable is found to be very small relative to its mean value, it indicates that this parameter has very little contribution to achieving the range of performance, and it is then taken as a common platform parameter. *On the other hand, the set of design parameters that have significant variation in the result cannot be held common for the family and is used as the set of non-platform variables (*i.e. scale factors) in the second stage of the VBPDM. Some mathematical details of formulating the compromise DSP are now given.

In Figure 2, the mean and standard deviation of design variables across all the products are represented as μ_{x_k} and σ_{x_k} ($k = 1, 2, \ldots, n$), respectively. In the same figure, $h_{ij}(\mathbf{X})$ represents the function value of the ith performance of the jth product ($\mathbf{X}$ is the vector of design variables). The desired mean and standard deviation of the ith performance across all the products in the family are represented by μ_{b_i} and σ_{b_i} whereas μ_{h_i} and σ_{h_i} are the actual mean and standard deviation of the ith performance. A *critical element of the proposed approach is to match the resulting performance distributions (captured by μ_{h_i} and σ_{h_i}) caused by the deviations of design variables, with the desired range of design requirements (represented by μ_{b_i} and σ_{b_i}).* The modeling of this part depends on whether a design requirement is considered as a constraint or a goal, and whether the desired value or limiting value is the same or different for all the products. As shown in Eqs. (2.1) through (2.6), the constraints/goals for meeting a range of performance are classified into six different categories that include:

a. Equality constraints on performance with dierent desired values from product to product,
b. Equality constraints on performance with the same desired value from product to product,
c. Inequality constraints on performance with dierent limiting values from product to product,
d. Inequality constraints on performance with the same limiting value from product to product,
e. Goal targets on performance, with dierent values for each product in the family, and
f. Goal targets on performance, with the same target value from product to product.

For category (a) design requirements, two sets of equality constraints are imposed to achieve the mean location and the dispersion of the performance (Eq. (2.1)). The modeling of category (b) requirements is identical to category (a) but with the dispersion set as 0 because the desired values of all of the equality constraints are the same in this case (Eq. (2.2)). For category (c) design requirements, only the mean performance is modeled (Eq. (2.3)). When the limiting values of all the products are the same in category (d), the worst case among all of the products is identified to satisfy the constraint (Eq. (2.4)). The evaluation of the worst case can be done through Taylor series approximation, statistical data sampling, or optimization techniques (Du and Chen [5]). For design requirements that are considered as goals (categories (e) and (f)), either the target or the mean of the different targets are modeled (Eq. (2.5) and (2.6)). The distribution of the targets is not important because goals represent the designer's wishes, and the targets are used to express the aspiration levels but not necessary the true levels of performance. Finally, Eq. (2.7) is used to model the commonality goal. The evaluations of the mean and standard deviation of performance responses can be implemented based on data sampling or a simplified approach such as Taylor series expansion.

2.2 Stage 2: Product Platform Instantiation

Once the common platform parameters and their values have been determined in the first stage of the VBPDM, values of the non-platform variables are sought to best satisfy the functional requirements of the individual products during the second stage of the VBPDM. One compromise DSP is formulated for each individual product in the family to optimize its non-platform variables. In each of these compromise DSPs, the settings of the common platform parameters identified from the first stage are known. The values of the non-platform design variables (*i.e.* scaling factors) must be found. The constraints and goals are appropriately modeled to satisfy the functional requirements specified for a particular product in the family. This process is also referred to as the instantiation of the product platform to yield the product family.

To demonstrate the approach, an example is adapted from Simpson *et al.* [23, 24] involving the design of a family of universal electric motors. In Simpson *et al.* [23, 24], a universal electric motor family is developed using the Product Platform Concept Exploration Method (PPCEM). The PPCEM-based motor family is used here as the benchmark for comparing the product family developed using the proposed VBPDM.

3 EXAMPLE PROBLEM

The fundamentals of universal electric motor operation and performance are described in detail in Simpson *et al.* [23, 24] along with the mathematical model used to analyze the performance of each motor. The requirements of the product family are described as follows. The design objective is to develop a family of ten universal electric motors ($j = 1, \ldots, 10$) to satisfy a range of torque (T_j) requirements. The other motor performance parameters are the masses of the motors (M_j), their efficiencies (η_j), their power (P_j), and their magnetizing intensities (H_j). In the following, issues related to these performances measures are discussed.

1. Magnetizing intensity (H_j): The design of the motor should ensure that the magnetizing intensity be below 5000 [Ampere · turns/m]. This is to ensure that the magnetizing flux within the motor does not exceed the physical flux carrying capacity of the steel.
2. Power (P_j): The desired power for each motor in the family is 300 W.
3. Eciency (η_j): The goal target for maximizing efficiency for all the motors is 70%.
4. Mass (M_j): A maximum allowable mass of 2.0 kg is assumed to define a feasible motor. The goal target for minimizing mass for all the motors is 0.5 kg.
5. Torque (T_j): The desired torque requirement for the ten motors are given by the set {0.05, 0.1, 0.125, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5} N.m.

Each motor has eight design variables that need to be determined during the design process for satisfying the needs and requirements of the product; they are (for $j = 1, \ldots, 10$):

1. Number of turns of wire on the motor armature, N_{c_j}
2. Number of turns of wire on each field pole, N_{s_j}
3. Cross-sectional area of the wire used on the armature, A_{wa_j}
4. Cross-sectional area of the wire used on the field poles, A_{wf_j}
5. Radius of the motor, r_j
6. Thickness of the stator, t_j
7. Current drawn by the motor, I_j
8. Stack-length of the motor, L_j

The current, I, drawn by each motor is treated as a state variable in the system, *i.e.* it is the amount of current which is drawn by the motor such that the given torque and power requirements are met for a given motor configuration.

3.1 Designing the Common Product Platform for the Motor Family

The first stage in the VBPDM is to design the product platform for the motor family. Following the discussed in Section 2.2.1, the compromise DSP formulation for designing the motor family platform is shown in Figure 3. The torque requirements of the motor family are modeled as equality constraints on performance with different desired values from product to product (category (a), Eq. (2.1)). The constraints on the magnetizing intensity (H) and mass (M) are modeled as inequality constraints on performance with the same limiting value from product to product (category (d), Eq. (2.4)). The constraint on power (P) is formulated as an equality constraint on the performance with the same desired value for all the products (category (b), Eq. (2.2)). The constraint ($\sigma_{h_{\text{power}}} = 0$), as per the generic formulation, is not included because the current, I, being a state variable, is used to satisfy the constant power requirement for all the motors without requiring an additional constraint. The mass and efficiency goals are incorporated into the compromise DSP as goals on performance with the same target value from product to product (category (f), Eq. (2.6)). The commonality goal is to minimize the normalized standard deviation of the system variables with a target of zero. The normalization value is approximated at the middle of the specified bounds for the standard deviations in the optimization process. The extent to which the commonality goal is achieved for a particular system variable gives an indication of whether or not the system variables can be held constant (*i.e.* made part of the product platform) within the product family. The goals on efficiency, mass, and commonality are assigned equal weights. *The weights can be varied suitably to represent designer preference for the variety and standardization requirements. The resulting product family is different when the weights assigned to the different goals are changed.* The reader is referred to Martin and Ishii [9, 10] and Gonzalez-Zugasti *et al.* [6] for metrics for tradeoff analysis when designing a product family.

The mean of the performance is assumed to be at the mean of the design variables. The standard deviation of the performance is calculated using the Taylor series approximation.

Thus for a general function:

$$h = f(x_1, x_2, \ldots, x_k) \tag{3.1}$$

the mean of h is:

$$\mu_h = f(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_k}) \tag{3.2}$$

Given
Al information necessary for assessing the achievement of goals and constraints (*i.e.* constants, mathematical relationships, goal targets)
Find
The mean and standard deviation of the design variables:

- Number of turns on the armature, N_c
- Number of turns on each pole on the field, N_s
- Area of he wire on the armature, A_{wa}
- Area of the wire onthe field, A_{wf}
- Thickness of the stator, t
- Radius of the motor, r
- Stack length, L
- State variable: Current drawn by the motor, I

Satisfy
The *system constraints* that must be satisfied for feasible solution:
Equality constraints on torque with different desired values fromproduct to product;

$$\mu_{h_{\text{torque}}} = \mu_{b_{\text{torque}}} \text{ and } \sigma_{h_{\text{torque}}} = \sigma_{b_{\text{torque}}} \text{ where } \mu_{b_{\text{torque}}} = 0.2325, \sigma_{b_{\text{torque}}} = 0.13675$$

Equality constraints on power with the same desired value for all products in the family;

$$\mu_{h_{\text{power}}} = \mu_{b_{\text{power}}} \text{ where } \mu_{b_{\text{power}}} = 300$$

Inequality constraints on mass and magnetic intensity (H);

$$\mu_{h_{\text{mass}}} \leq 2\,\text{kgs and } \mu_{h_H} \leq 5000$$

The *system goals* that must achieve a specified target as far as possible:

Goal targets for mass and efficiency with the same target value from product to product;

$$\mu_{h_{\text{mass}}}/0.5 + d_I^- - d_I^+ = 1 \text{ and } \mu_{h_\eta}/0.7 + d_2^- - d_2^+ = 1$$

Commonality goal:

$$(\sigma_{N_c}/a_{N_c} + \sigma_{N_s}/a_{N_s} + \sigma_t/a_t + \sigma_r/a_r + \sigma_L/a_L)/5.0 + d_3^- - d_3^+ = 0$$

Minimize

$$Z = \sum_{t=1}^{3} \mathrm{W}_t(\mathrm{d}_t^- + \mathrm{d}_t^+);\ \mathrm{W} = 0.333$$

FIGURE 3 Compromise DSP for determining the motor family platform.

and the standard deviation of h is:

$$\sigma_h = \sqrt{\left(\frac{dy}{dx_1}\right)^2 \sigma_{x_1}^2 + \cdots + \left(\frac{dy}{dx_k}\right)^2 \sigma_{x_k}^2} \tag{3.3}$$

where μ_{x_k} is the mean of the kth design variable, and σ_{x_k} is the standard deviation of the kth design variable.

It should be noted that, for better accuracy, we could use sampling methods such as Monte Carlo simulation [5] could be used; however, since the deviation of torque requirements is not large, the Taylor series approximation is sufficiently accurate. The results of solving the compromise DSP formulation given in Figure 3 are tabulated in Tables I and II, in which the identified mean and the standard deviation of the design variables and resulting perfor-

TABLE I Identified mean and standard deviation of design variables

Design Variable	*Mean*	*Standard deviation*	*Ratio*
A_{wa} [mm^2]	0.280	0.0	0.0
A_{wf} [mm^2]	0.384	0.0	0.0
t [mm]	7.15	1.74	24%
r [cm]	2.38	1.13	47%
N_c	1120	122	10%
N_s	72	6	8%
I [A]	3.66	–	
L [cm]	2.51	0.17	6%

TABLE II Performance parameters for the product family

Response	*Mean*	*Standard deviation*
Power [W]	300	–
Efficiency	0.71	–
Mass [kg]	0.72	–
Torque [N-m]	0.2235	0.1354
H	4168	–

mance variations are provided, respectively. The different degrees to which the *commonality goal* is satisfied by the different system variables provides an indication about the system variables that should compose the product platform. The decision on how much variation is negligible is problem specific; however, the value of the standard deviation as a percentage of the mean value can provide a good indication. For this problem, standard deviations that are less than 10% of the mean value are considered to be small enough for the corresponding design variable to be considered a common platform parameter.

Based on the results in Table I, the product platform is formed comprising the armature wire area (A_{wa}), field wire area (A_{wf}), the number of turns of field wire (N_s), and the stack length of the motor (L). The radius of the motor (r), thickness (t), number of turns of the armature wire (N_c), and current (I) are allowed to vary from motor to motor within the product family. From Table II, the power requirement is satisfied for each product at $\mu_{\text{power}} = 300\text{W}$ and $\sigma_{\text{power}} = 0$. The product platform also satisfies the target for the mean efficiency; however, the target of 0.5 kg for the mean mass is not achieved. The constraint on mean torque is satisfied with a violation of 3.8%, which is within the specified tolerance range. The constraint on standard deviation of the torque is better satisfied with a violation of only 0.9%. The constraint on the magnetic intensity is also satisfied at this stage of the design.

3.2 Instantiation of the Individual Products in the Motor Family

The next stage of the VBPDM is to instantiate the individual motors of the product family using the specifications for the common parameters that describe the product platform. The compromise DSP formulation for designing the individual motors is given in Figure 4. While the common platform parameters determined from Stage 1 are fixed as constant parameters, the to-be-identified variables are the four remaining non-platform variables: number of turns on the armature, thickness of the stator, current drawn by the motor, and radius of the motor. The constraints and goals are related to the design requirements origin-

Given: Platform settings for A_{wa}, A_{wf}, L, and N_s (Table I)

Find: **(Solved once for each of $j = 1, \ldots, 10$)**

- Number of turns on the armature, N_{c_j}
- Thickness of the stator, t_j
- Current drawn by the motor, I_j
- Radius of motor, r_j

Satisfy:

The system constraints:

- Magnetizing intensity, $H_j \leq 5000$
- Power, $P_j = 300\,W$
- Mass, $M_j \leq 2.0\,kg$
- Individual torque requirements

$$T_j = \{0.05, 0.1, 0.125, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5\} N - m$$

The *system goals* that must achieve a specified target as far as possible: Goal targets for mass and efficiency

$$M_j/0.5 + d_1^- - d_1^+ = 1 \quad \text{and} \quad \eta_j/0.7 + d_2^- - d_2^+ = 1$$

Minimize

$$Z = \sum_{t=1}^{2} W_t(d_t^- + d_t^+);\ W_t = 0.5$$

FIGURE 4 Compromise DSP for instantiating the motor platform.

ally stated at the beginning of Section 3. Note that the commonality goal is not utilized during the second stage of the VBPDM since the product platform has already been determined. The product family thus developed, represented by the values of all design variables and the resulting performance, is listed in Table III.

4 VERIFICATION

The proposed approach is verified from two aspects. First, the results obtained using the VBPDM are compared to those from the PPCEM [23] where the common platform variables are pre-specified. This is to show that by choosing the best set of common platform variables

TABLE III Universal motor product family instantiations by the vbpdm

	Product Specifications (Design Variables)								*Responses*			
Motor	*Nc*	*Ns*	*Awf [mm^2]*	*Awa [mm^2]*	*I [A]*	*r [cm]*	*t [mm]*	*L [cm]*	*T [N-m]*	*P [W]*	*η*	*M [kg]*
1	374	72	0.280	0.384	2.92	2.32	4.05	2.51	0.05	300	.89	.50
2	731	⇓	⇓	⇓	3.16	1.99	3.19	⇓	0.1	⇓	.82	.50
3	906	⇓	⇓	⇓	3.29	1.83	2.85	⇓	0.125	⇓	.79	.50
4	1068	⇓	⇓	⇓	3.42	1.70	2.69	⇓	0.15	⇓	.76	.50
5	1183	⇓	⇓	⇓	3.65	1.82	2.97	⇓	0.2	⇓	.71	.57
6	1260	⇓	⇓	⇓	3.85	1.95	3.79	⇓	0.25	⇓	.67	.63
7	1295	⇓	⇓	⇓	4.06	2.06	3.92	⇓	0.3	⇓	.64	.67
8	1297	⇓	⇓	⇓	4.29	2.16	4.00	⇓	0.35	⇓	.60	.72
9	1297	⇓	⇓	⇓	4.49	2.27	4.37	⇓	0.4	⇓	.58	.76
10	1269	⇓	⇓	⇓	4.89	2.49	4.99	⇓	0.5	⇓	.53	.83

instead of using the pre-specified ones, the present approach provides more flexibility when designing the platform and is therefore expected to yield better overall performance of the product family. Second, the mathematical structure of the example problem is studied in more detail to verify whether or not the results obtained from the proposed approach are consistent with the physics of the problem.

4.1 VBPDM *vs*. Other Approaches

In Simpson *et al.* [23], the motor stack length (L) is pre-selected as the scale factor around which the product family is derived. The variation of the stack length is expected to result in the desired variation of the motor torque while the other design variables form the common product platform. We believe that the approach described in this paper provides decision support for selecting the appropriate common parameters for the product platform, removing any trial-and-error that may occur.

A comparison of the product family developed using the PPCEM by selecting the stack length as the scale factor, with the product family developed using the VBPDM is presented in Table IV. To improve the product family obtained by the PPCEM, Simpson *et al.* [23] also developed another product family with reduced standardization, by assuming a platform of constant radius of motor (r), and constant thickness (t). Table V contains the comparison of this product family with the VBPDM family.

All the motor families in Tables IV and V meet their goals for both power and torque; however, the values for efficiency and mass differ. Hence, we can use these two parameters to compare the product families against one another. Tables IV and V list the percentage difference of each response from the VBPDM family to the PPCEM family. For efficiency, a positive change denotes an improvement from the PPCEM family to the VBPDM family, while a negative change denotes an improvement in the mass. A motor which has achieved its target mass (0.5 kg) and efficiency (70%) is considered equivalent to a motor with a mass which is lower than the target or an efficiency which is higher than the target. From Tables IV and V, it is observed that the product family obtained using the VBPDM is significantly better than the PPCEM Family 1 in terms of achieving both the mass and efficiency goals, on average. Results in Table V indicate that the VBPDM is also better than the PPCEM Family 2 on average mass and efficiency. With the VBPDM, it is possible to achieve a high degree of

TABLE IV Comparison of the performance between the VBPDM motors and the PPCEM motor family with only stack length varying

	VBPDM Family		*PPCEM Family 1*		*Percent difference*	
Motors	η	*M[kg]*	η	*M[kg]*	η	*M[kg]*
1	.89	.50	.768	.380	Equiv.	Equiv.
2	.82	.50	.722	.520	Equiv.	− 3.9
3	.79	.50	.700	.576	Equiv.	− 13.2
4	.76	.50	.679	.625	+ 11.9	− 20.00
5	.71	.57	.639	.703	+ 11.11	− 18.92
6	.67	.63	.602	.759	+ 11.30	− 17.00
7	.64	.67	.568	.797	+ 12.68	− 15.93
8	.60	.72	.536	.820	+ 11.94	− 12.20
9	.58	.76	.505	.830	+ 14.85	− 8.43
10	.53	.83	.448	.820	+ 18.30	+ 1.22
			Average change:		+ 13.15	− 12.04

TABLE V Comparison of the responses between the VBPDM motors and the PPCEM motor family with a platform of constant radius and thickness

	VBPDM Family		*PPCEM Family 2*		*Percent difference*	
Motors	η	*M[kg]*	η	*M[kg]*	η	*M[kg]*
1	.89	.50	.747	.397	Equiv.	Equiv.
2	.82	.50	.721	.456	Equiv.	Equiv.
3	.79	.50	.711	.477	Equiv.	Equiv.
4	.76	.50	.701	.499	Equiv.	Equiv.
5	.71	.57	.675	.568	+ 5.19	+ 0.3
6	.67	.63	.646	.646	+ 3.72	− 2.48
7	.64	.67	.622	.712	+ 2.89	− 5.9
8	.60	.72	.599	.774	+ 0.17	− 6.9
9	.58	.76	.577	.833	+ 0.52	− 8.76
10	.53	.83	.538	.941	− 1.49	− 11.8
			Average change:		+ 1.83	− 5.9

commonality while providing sufficient variety by properly selecting the common platform parameters.

The three families of products can also be compared graphically by plotting the corresponding mass and efficiency of each motor within each family as shown in Figure 5. In this manner, it is easy to determine which motors have achieved the desired targets of 70% for efficiency and 0.5 kg for mass. Figure 5 shows that 4 of the 10 motors in the VBPDM family achieve the desired performance targets while only 1 motor from PPCEM Family 1 achieves the desired performance targets. Meanwhile, 4 of the 10 motors from PPCEM Family 2 also meet the desired performance targets; however, the efficiency and mass of the other six motors are all worse than those of the corresponding motors in the VBPDM family except the mass for Motor 5. *The flexibility of the VPBDM has enabled us to find an equivalently good family of motors without the trial-and-error process used*

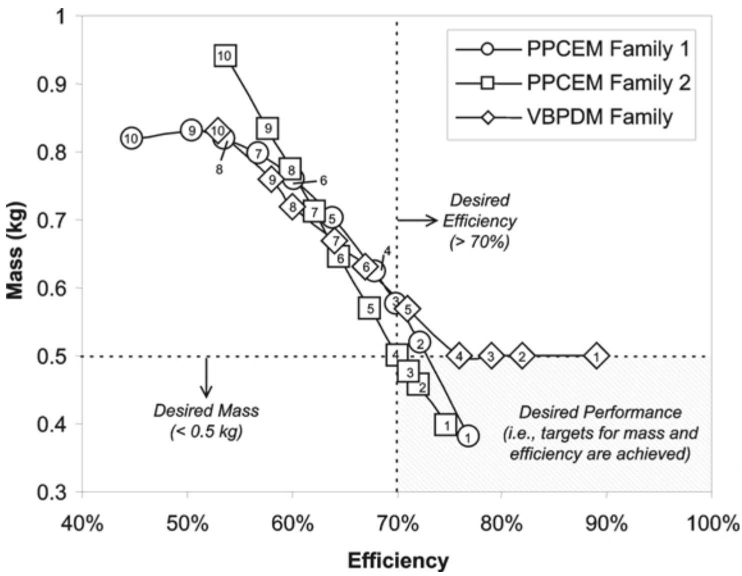

FIGURE 5 Graphical comparison of VBPDM and PPCEM motor families.

in selecting common platform parameters for the PPCEM families. This flexibility helps us determine the best compromise between product platform commonality and individual product performance within a product family described by a range of performance requirements.

4.2 Verification Based on Model Behavior

To gain a better understanding of the physical behavior of the example problem, response surface models were developed (see, *e.g.* Myers and Montgomery [16]) of the different motor performance using the OPTIMUS® software. The response surface models are developed for the torque (T), power (P), efficiency (η), and mass (M) of each motor in the neighborhood of the optimum solution obtained by the VBPDM (see Table I). Each response surface is developed for a small range of $\pm 5\%$ around the mean values for the design variables in Table I.

The contribution plots shown in Figure 6 indicate the importance of the different design variables on each performance output. The coefficients are calculated in a normalized design

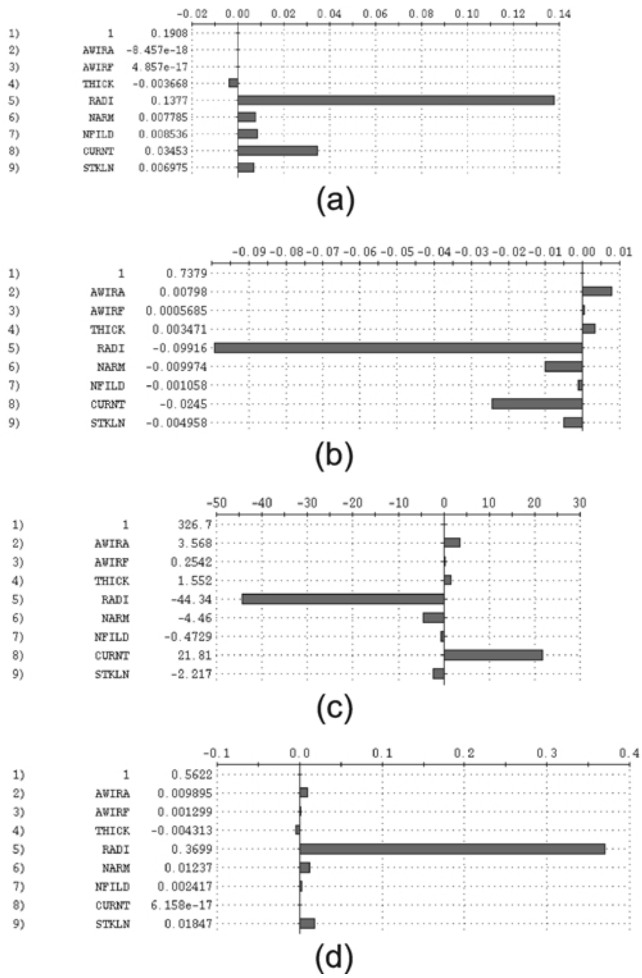

FIGURE 6 Contributions of different variables for (a) torque, (b) efficiency, (c) power, and (d) mass.

space, so that they can be compared. The contribution plot for torque (see Figure 6a) shows that the radius (r) is the most dominant factor. This matchs with the deviation of variables obtained from the VBPDM in Stage 1 (see Table I), which indicates that the radius (r) of the motor has the greatest contribution towards satisfaction of the range of performance requirements. Note in this case that torque is the only performance for which the standard deviation has to meet the target for achieving a range of performance within the product family. However, in general cases, the variation of the different design variables obtained by the VBPDM for satisfying the range of performance requirements depends on the multiple competing objectives. A study of contribution plots of the different performance measures (Figures 6a-d) indicates the changing importance of the variables in the different responses.

It should be noted that the maximum contributing variable for a particular performance measure is not an obvious or automatic choice for satisfying the range of performance when we have competing objectives. Therefore, *sensitivity analysis is not sufficient enough to identify what should be the common platform parameters or non-platform variables within a given product family.* The platform design optimization process in Stage 1 is needed to find the best set of design parameters to vary, considering all of the objectives and the desired range of performance.

5 CONCLUSIONS

In this paper, the Variation-Based Platform Design Method (VBPDM) is proposed as a logical extension of the PPCEM, providing a more flexible formulation for modeling a product family to identify the common platform parameters and the non-platform variables (scale factors). This formulation enables a better tradeoff between product platform commonality and the performance of the individual products within the product family. The VBPDM uses a two-stage approach to design and develop a product platform and corresponding family of products. In the first stage, the VBPDM uses the compromise DSP to make tradeoffs between commonality and individual product performance within the product family to identify the product platform defined by the common platform parameters. Variation-based modeling is used to obtain a range of solutions for the corresponding range of performance requirements for the product family. This ranged set of solutions represents the deviation of the design variables across the different products in the product family. The compromise DSP formulation is used to demonstrate how one optimization formulation can be used to model the needs for a range of products using variation-based representations; however, alternate formulations can be envisioned. In the second stage of the VBPDM, the common product platform is used as the core to develop the individual products of the family, *i.e.*, determine the values of non-platform variables.

A family of ten universal electric motors with a range of torque requirements is used to illustrate the proposed method. The product family developed using the PPCEM (Simpson *et al.* [23]) with pre-specified platform variables is used for comparison to discuss the advantages of the present approach. Then, response surface models are used to study the effect of the contributing design variables to confirm further the validity of the results obtained. These verification studies illustrate the advantages of the proposed method as well as the potential challenges associated with finding good product platforms due to the existence of multiple equivalent solutions.

The compromise DSP formulation can also be used to model the cost benefits of making different variables common, thereby making the tradeoff truly reflect the cost of providing variety. To evaluate the benefits of providing variety versus commonality, a measure of customer preference for variety also needs to be incorporated in the model in addition to manufacturing costs and related considerations. Finally, while most product family design strategies and methods advocate developing modular products, this investigation of a method for platform scaling is suitable for even products having complex integral architecture. Further research is required to extend the applicability of the method for designing large systems such as an automobile or aircraft that involve many sub-systems.

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