A Probabilistic-Based Design Model for Achieving Flexibility in Design

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ABSTRACT

In this paper we propose the use of a probabilistic-based design model as a basis for providing the flexibility in a design process that allows designs to be readily adapted to changing conditions. Our proposed approach can be used to develop a range of solutions that meet a ranged set of design requirements. Meanwhile, designers are allowed to specify the varying degree of desirability of a ranged set of design performance based on their preferences. The Design Preference Index (DPI) is introduced as a design metric to measure the goodness of flexible designs. Providing the foundation to our work are the probabilistic representations of design performance, the application of the robust design concept, and the utilization of the compromise Decision Support Problem (DSP) as a multiobjective decision model. A two-bar structural design is used as an example to demonstrate our approach. Our focus in this paper is on introducing the probabilistic-based design model and not on the results of the example problem, *per se.*

Key words: Flexible Designs, Ranged Set of Design Requirements, Probabilistic-Based Design Model, Design Preference Index, Robust Design, Decision Support.

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NOMENCLATURE

d_i^-, d_i^+	Deviation variables in the compromise DSP
DPI	Design Preference Index
f_i (y_i)	Probability density function of design performance
F_i ($\mathbf{x}$)	Functional relationship between performance and design variables
g_i ($\mathbf{x}$)	System constraint function
g_i *($\mathbf{x}$, $\Delta\mathbf{x}$)	Worst possible value of the constraint under deviations
m	Number of system goals
n	Number of design variables
P_i (y_i)	The preference function of design performance
q	Number of system constraints
x_i	Design variable
x_i^L, x_i^U	Lower bound and upper bound of x_i
Δx_i	Range of design solution
Δx_{imin}	Minimum allowable deviation of design solution
Δx_i^L, Δx_i^U	Lower bound and upper bound of Δx_i
y_i	Design performance or attribute
Δy_i	Deviation of design performance
Δy_{imax}	Maximum allowable deviation of design performance
Z_i (d_i)	Function of deviation variables to be minimized

1. INTRODUCTION

To achieve potential time savings in a design process, the decisions made in the early stages of design must be comprehensive yet *flexible* enough to allow designs to be readily adapted to changing conditions. In this paper, we address two specific issues related to flexibility in design. These two issues are (1) flexibility in specifying the design requirements and (2) flexibility of the design solutions. This is further explained as follows:

- First, early in the product development, the level of performance that will be judged as desirable or acceptable may be uncertain. When applying conventional optimization techniques, designers have to pick precise numbers to specify design requirements, such as the use of the minimum level of a performance in constraints. In the lack of representations of uncertainties in a design model, designers typically choose to be conservative. An example is the use of factors-of-safety in stress analysis. As a result, overdesign is utilized to compensate the risk caused by the uncertainty. This raises the question: *How can we capture this uncertainty and offer flexibility in specifying the design requirements* so that the designs marginally outside the precise minimum level of performance are not worthless.
- Second, considering that the decisions made in the early stages of design are used to define the design requirements for the later development, to enhance the flexibility of a design process, these decisions are desired to be a range

instead of a single point solution. This is consistent with the notion of looking for a set of "satisficing" (equivalently good) solutions (Simon, 1982) in a complex real world problem. It is therefore necessary to seek an alternative representation of design variables as well as to consider the means of evaluating the performance variation resulted from a range of solutions.

Conventionally, the design requirement could be considered as either a rigid constraint or a soft objective if the problem is solved using optimization techniques. Using "A(**x**)" to denote a function of achievable performance versus design variables **x**, a rigid case could be stated as "A(**x**) must be greater or smaller than, or equal to B (a scalar)". A soft case could be stated as "A(**x**) needs to be minimized or maximized as close as possible to B". In all cases, B is picked as a precise value to distinct designs that are desirable or undesirable. The conventional approach provides a rigid framework for expressing design preferences that can be considered singularly undesirable, however, little flexibility in expressing different degrees of desirability. Work has been done in recent years to incorporate designers' preferences with degrees of satisfaction (desirability) in specifying design requirements. Thurston (1991) use utility theory (von Neumann and Morgenstern, 1947) based *preference functions* to express designers' preferences over single or multiple attributes. Wallace et al. (1996) define *specification functions* to indicate the subjective probability that performance levels are acceptable. Mohandas and Sandgren (1989) recommend to use fuzzy goals to model the degree of satisfaction level. Messac (1996) develops an approach called physical programming that utilizes the *aggregate preference function* which reflects the preferences expressed in the class function of each attribute.

Yoshimura and Kondo (1995) develop *satisfactory functions* to express the satisfactory levels of both the manufacturing engineers and the product designers. Though the aforementioned approaches differ in the conditions (e.g., types of uncertainties) under which they are used, a common feature is the use of a *function* to express different degrees of designers' satisfaction versus the level of a performance attribute.

The methods employed in the literature for modeling the variations of design parameters (or variables) can be classified into four categories: fuzzy set theory (Wood and Antonsson, 1987), interval methods (Chen and Ward, 1995), sensitivity analysis methods (Sandgren et al., 1985), and probabilistic-based methods such as robust design (Sundaresan, et al., 1993; Parkinson, et al., 1993). The difference between the fuzzy set theory and probabilistic methods for dealing with different types of uncertainties are discussed by Wood and Antonsson (1990) and will not be repeated here. With interval methods, each design parameter can be specified as a range or an interval, however, the method is often limited to linear relationships, a condition could seldom be satisfied in complex systems design. While sensitivity analysis is focused on reducing the local rate of change of design performance, the robust design or Taguchi method (Phadke, 1989; Taguchi, 1993) goes one step further by introducing uncertainty or noise in the system and generating optimal solutions that could reduce the impact of the uncertainty in a global scale. In the engineering design community, the robust design concept has been extended into the early stages of design to improve the "quality" of design decisions. A part of the author's investigation (Chen et al. 1996) is to extend the robust design concept into the early stages of design for making decisions that are robust to the changes of downstream design considerations (called Type I robust design), and decisions that are flexible to be allowed to vary within a range (called Type II robust design). What is relevant to this work is the Type II robust design in which a range of solutions are modeled by the variations of design variables.

Corresponding to the variations of design variables, a design metric is required to measure the goodness of fluctuating performance. It is interesting to note that Suh has brought the notion of flexibility into his information axiom by considering both the design performance as achievable performance *range* and the desirable level of performance as *target range*. The metric for *information content* is defined in the following equation:

$$I = \text{Log (system range/common range)}. \qquad (1)$$

In Eqn. (1), Suh relates the information content to the probability of success of achieving the specified functional requirements. The *system range* is the *achievable performance range* of the system. The *common range* is the *overlap between the achievable performance range of the system and the target range for the system.* By information axiom, a good design with minimum information content corresponds to the design with the largest possible overlap between the achievable performance range and the target range. Though information content is claimed to be a probability measure of success, the probability density function of system range and target range are all assumed to be *uniform***.** This representation is not sufficient for comparing designs with different performance variation behaviors but the same system range, or when the performance within the target range assumes different degrees of desirability.

We conclude from the preceding review that most of the current design methodologies are still supported by the conventional modeling of optimizations, in which only a single point solution is sought. Even in some work where variations of design performance caused by the deviations of design parameters are considered, little has been done to provide a range set of solutions which meet a ranged set of design requirements. Though quantitative inference method based on interval arithmetic has been utilized to develop a range of solutions, they are generally not applicable for complex designs involving closed-form analysis due to the mathematical difficulties. On the other hand, preference functions of performance levels are introduced to represent degrees of satisfaction (desirability) in specifying design requirements, however, few work has addressed the issue of developing flexible design solutions under this condition. The aim of this work is to address these issues simultaneously through a generic design model for determining a range of design solutions while incorporating designers' preferences.

2. THE PROBABILISTIC-BASED DESIGN MODEL

In this work, we propose a probabilistic-based design approach which is believed to be applicable for problems involving computationally expensive and closed-form (inexplict) analyses. The Design Preference Index (DPI) is introduced as a design metric to measure the goodness of flexible designs. The concept of robust design is used to control the influence that variation in design variables has upon the range of performance. An important aspect of this work is to permit a designer to independently address the needs associated with each design requirement. The compromise Decision Support Problem (DSP) is used as a multiobjective mathematical construct (Mistree, et al., 1993) that enables a designer to determine the values of design variables which satisfy a set of constraints and achieve a set of goals.

2.1 Preference Functions of Performance Levels

In conventional design optimizations, a precise number is often picked to distinct designs that are desirable or undesirable. In some cases, it is hard for designers to pick a value to make such a distinct evaluation. To capture varying degrees of preference on the levels of performance, a preference function of performance levels is employed in our model. In general, a preference function of performance levels P(y) is a function defining the relationships between the degree of desirability P and the level of performance y. The function is defined in the range of 0 and 1. A value of 1 represents full preference, any design with desirability as 1 can be deemed as fully acceptable or desired; a value of 0 represents no preference, which could be considered as unacceptable.

A comparison is made in Figure 1 to show that when the preference functions are used for expressing smaller-the-better (STB), larger-the-better (LTB), and center-the-better (CTB), they are more flexible compared to the way used in conventional optimizations. Note, the preference function can have various types of form and is not limited to a linear function.

Insert Figure 1 Preference Functions in the Proposed Model

The structure of the preference function of the performance levels seems to be similar to that of the utility function (von Neumann and Morgenstern, 1947). However, "utility" has a very specific connotation, as defined by von Neumann and Morgenstren (1947) and Keeney and Raiffa (1976). Utility functions must obey a whole set of conditions and axioms, while the mathematical structure of a preference function for performance levels considered in this work is more flexible. For example, a preference function does not need to be monotonically increasing or decreasing, a feature of the utility function. In the decision science literature, utility function is used to capture a designer's preference of one design over the other for single or multiple performance attributes, and is often used under the condition of risk. The proposed *preference function of performance levels* represents designer's subjective judgement of "degree of desirability" for each level of a single performance attribute. Note this concept differs from *preference functions* used in the decision science literature which correspond to the ordering of design preferences.

2.2 A Range of Design Solutions

To enhance system flexibility, we are interested in developing a design model that could find a range of solutions for design variables instead of a point solution. When the deviation of design solutions is considered, the resulting performance will correspondingly vary within a range. In this work, the uncertainties associated with which solution obtained from the early stage will be used for the later development are modeled by the probabilistic distributions of design variables. Under this notion, Monte Carlo simulations or other statistical techniques such as Design of Experiments are extremely useful to evaluate the performance distribution. Figure 2 is an example to illustrate the distribution of the performance y caused by the deviations of two independent variables $\bar{x}_1 \pm \Delta x_1$ and $\bar{x}_2 \pm \Delta x_2$. The probability density function f(y) describes the performance distribution when assuming random variations of designs in the box formed by the design ranges.

Insert Figure 2 Mapping between Design Variables and Design Performance

2.3 Design Preference Index (DPI)

To evaluate the goodness of a design when both the design performance and the preference level of performance vary within the ranges, Design Preference Index (DPI) is introduced. Mathematically, DPI is defined as the expected preference function value of design performance within the range of design solutions, as depicted in Equation :

$$DPI = E[\,P(y)\,] = \int_{\bar{y}-\Delta y}^{\bar{y}+\Delta y} P(y)f(y)dy \qquad (2)$$

This function is used as a measure to evaluate the goodness of varying performance in successfully satisfying a ranged set of design requirements. In Figure 3, we demonstrate three different designs that assume different values of DPI. In design (a), the preference function value P(y) remains 1 within the range $[\bar{y}-\Delta y,\ \bar{y}+\Delta y]$, while $\bar{y}$ is the average of performance. Then,

$$DPI = E[\,P(y)\,] = \int_{\bar{y}-\Delta y}^{\bar{y}+\Delta y} f(y)dy \approx 1 \qquad (3)$$

This is the desired situation, which means that design can fully meet the design requirements under the variation of performance. However, DPI = 1 cannot be achieved in some cases, as illustrated in design (b) and design (c). From this illustration, we could see that maximizing DPI as close as possible to 1 is desired. The idea of using the expected preference function value to measure the goodness of a design under variations is rooted in probabilistic optimization. The function in Eqn. 3 is also similar to the design metric used in the acceptability-based method (Wallace et al., 1996), by which design is assumed as a specification-satisfaction process. The difference in our work is that we are using the concept to evaluate a range of design solutions, whose value is chosen at a random basis.

Insert Figure 3 Design Preference Index

2.4 Trade-offs Between Multiple Design Requirements

A multiobjective decision making model, called the compromise Decision Support Problem (Mistree et al., 1993, Figure 4), is used in this work to make the trade-offs between the multiple design requirements. The compromise DSP is a multiobjective mathematical construct which is a hybrid formulation based on mathematical programming and goal programming (Charnes & Cooper, 1961). The method is superior to conventional optimization methods in its capability of handling multiple design objectives at either the same or different priority levels. In Figure 4, goals may either be weighted in an Archimedean solution scheme or, using a preemptive approach, rank-ordered into priority levels to effect a solution on the basis of preference. The objective is to minimize the deviations of different goals $A_i(\mathbf{x})$ from target values G_i using lexicographic minimization (Ignizio 1985), a formula comprised of deviations variables d_i^-, d_i^+.

Our proposed probabilistic-based design model, Figure 5, is constructed based on the conventional compromise DSP by including the probabilistic representations of performance variations, and the multiple goals on maximizing the design metrics DPIs as close as possible to 1. The major differences between the conventional compromise DSP (Figure 4) and the probabilistic-based design model (Figure 5) are discussed as follows:

- In the section "Find", in addition to the nominal value of design variables $\bar{x}_i$ and the deviation variables $[d_i^-, d_i^+]$, the variations of each variable, Δx_i are also taken into account as design variables in the probabilistic-based design model. Once the $\bar{x}_i$ and Δx_i are found, the range of design variables can be defined as $[\bar{x}_i - \Delta x_i, \bar{x}_i + \Delta x_i]$.
- In the section "Satisfy", design requirements are split into two parts as "system constraints" and "system goals". System constraints (Eqns. (4)-(6)) are those design requirements that have to be satisfied within a specific limit in all circumstances, while system goals Eqn. (7) are those design requirements whose desirable targets are defined by the preference function P(y). In the group of system constraints, the lower limit on the deviation of design variables (Δx_i) is introduced to ensure the minimum flexibility of design solutions (Eqn. (5)). Without this constraint, the range of a design variable, Δx_i, will otherwise tend to converge to zero to result in the minimum deviation of design performance. Eqn. (6) is an optional constraint on the maximum variation of performance under the robust design consideration.

- In the group of "system goals", the objectives on maximizing the Design Preference Indices defined in Eqn. (8) are modeled as multiple goals (Eqn. (7)). An important aspect of this work is to permit a designer to independently address the needs associated with each design requirement. This is accomplished by maximizing the DPI for each design requirement separately. The target value for the DPI associated with each design requirement is assigned as 1. The tradeoff between the multiple requirements is captured by the deviation function in Eqn. (12). With the preemptive deviation function, these goals would be placed on different priority levels.

Insert Figure 4 Mathematical Form of a Conventional Compromise DSP

Insert Figure 5 The Probabilistic-Based Multiobjective Design Model

For design requirements that are preferred to be treated as rigid constraints, we have the option of using the inequality constraint (Eqn. (4)), in which $g_i^*(\bar{\mathbf{x}}, \Delta\mathbf{x})$ represents the worst possible value of the ith constraint. In the worst case scenario, it is assumed that all variations of system performance may occur simultaneously in the worst possible combinations of design variables. This constraint is used to guarantee that the whole range of the design solutions $[\bar{\mathbf{x}} - \Delta\mathbf{x}, \bar{\mathbf{x}} + \Delta\mathbf{x}]$ satisfy the constraints. The use of the worst case scenario is more conservative than the use of probabilistic constraints (Eggert and Many, 1993). $g_i^*(\bar{\mathbf{x}}, \Delta\mathbf{x})$ can be obtained by evaluating the combinations of design solutions in the range based on the Design of Experiments (DOE) techniques (Box et al. 1978) or the first-order Taylor expansion when the design deviations are small.

3. DESIGN OF A TWO BAR STRUCTURE USING THE PROBABILISTIC-BASED DESIGN MODEL

3.1 The Conventional Design Model

A schematic picture of the two-bar structure system is included in Appendix, Figure A.1. The purpose of this design is to find the values of two design variables, i.e., x_1, nominal diameter of the cross section, and x_2, the height of the two-bar structure. Design requirements include the considerations on normal stress, buckling stress and the total volume. When applying the conventional optimization technique, single values will be determined as the solutions for x_1 and x_2. The design requirements could be stated as: (a) the normal stress S must be lower than the critical buckling constraint limit σ_{crit} , (b) the total volume V needs to minimized; value lower than 600,000 mm^3 is desired, and (c) the normal stress S needs to be minimized; value lower than 400 N/mm^2 is desired. Based on this problem statement, the design model in terms of the conventional compromise DSP could be represented as the one shown in Figure A.2. In this conventional model, design requirement (a) is modeled as a constraint, while requirements (b) and (c) are modeled as goals. Maximizing the values of these two goals is achieved by minimizing the deviation function values d_i^+, i=1, 2.

3.2 The Probabilistic-Based Design Model

The probabilistic-based design model illustrated in Figure 5 is applied to the development of flexible design solutions for the two-bar problem. To express designers' different degrees of satisfaction for the performance levels of volume and stress, Figures 6.a and 6.b are used as the preference functions for these two performance attributes, respectively.

Insert Figure 6 Preference Functions for Volume and Stress

To specify the desired minimum variation of design variables Δx_{imin}, we choose to use a certain percentage of the nominal value x_i , denoted by coefficient α. The maximum variation of the performance Δy_{imax} is specified by a coefficient β to the nominal value $\bar{y}$. Since there are only two design variables, $g_i^*(\mathbf{x}, \Delta\mathbf{x})$ is evaluated by 3-level full factorial combinations of design variables. In order to evaluate the DPI as defined in Eqn. (8), the performance distribution, f_i (y_i), needs to be first obtained based on the functional relationship $F(\mathbf{x})$ and the distribution of $\mathbf{x}$. Ideally, this could be implemented by the Monte Carlo simulations. However, when the computation time is too demanding, we could use the approximation to assume the performance follows normal distributions based on the central limit theorem (Hines and Montgomery 1990). When the variance Δy is approximated by $3\sigma_y$, the probability of the performance falling within the range $[\bar{y} - \Delta y, \bar{y} + \Delta y]$ is 99.7%.

3.3 Studies of the Solutions

Comparison of the set of solutions and the point solution

The probabilistic-based design model is solved while assuming the coefficients for defining the limits of variations of design variables and design performance are taken as α=10% and β_1=β_2=20%, respectively. The solutions are presented in Table 1. For comparisons, the single point solution derived from the conventional model (Figure A.2) is also provided. In our initial study, we use the Archimedean deviation function in both design models. With this function, the two goals (stress and volume) are placed at the same priority level with equal weights.

Insert Table 1 Results for the Two-Bar Problem

When using the probabilistic-based design model, solutions are in the form of [$\bar{\mathbf{x}} - \Delta\mathbf{x}$, $\bar{\mathbf{x}} + \Delta\mathbf{x}$] which composes a rectangle in the space of two design variables. From Table 1, it is noted that correspondingly, the values of design performance (Volume and Normal Stress) also vary within a range. The achieved DPI values for two goals are 0.84487 and 0.882679, respectively. When using the conventional model, the solution is a point. It is noted that it is a feasible solution and is embodied in the rectangular sets of solutions obtained from the probabilistic-based design model.

Impact of the lower limit on the deviation of design variables

To study the impact of this limit value, we take α as different values ranging from 0% to 10%. The ranges of design solutions are depicted by the rectangles in the contour plots as shown in Figure 7. In Figure 7, there are three groups of curves representing the locations of two goals, i.e., normal stress $S(x_1, x_2)$, volume $V(x_1, x_2)$, and the location of the design constraint, represented by buckling stress minus normal stress, i.e., $\sigma_{crit}(x_1, x_2) - S(x_1, x_2)$. The curves are plotted within the upper and lower bounds of the design variables x_1 and x_2. The values of contours are marked on the figure. For example, the value of the constraint ranges from 0 to 600 N/mm^2. Based on the design model, to be feasible, the value of $\sigma_{crit}(x_1, x_2) - S(x_1, x_2)$ cannot be less than 0, which indicates that the designs at the right hand side of the curve $\sigma_{crit}(x_1, x_2) - S(x_1, x_2)=0$ are feasible.

It is noted that when α decreases, the size of the rectangle becomes smaller. The nominal value could be roughly judged by the location of the center point (marked as #) of the rectangle in Figure 7. It is noted that when α decreases, the rectangles shrink towards the left upper corner of the feasible design space in all cases. When α=0% in Scenario IV, the solution can be considered as a single point as $\Delta\mathbf{x}$ are very close to 0. This study indicates that our proposed design model is a generic model. It can be used for developing a range of solutions in different phases of design when the desired degree of flexibility of design solutions is different. It can be as well used for developing a single point solution by setting α=0.

Insert Figure 7 Results for Illustrating the influence of a

Impact of weights and priority levels of goals

Our proposed approach is also very powerful for studying the tradeoffs between multiple design requirements. When Archimedean deviation function is used, designer's willingness in making tradeoffs between different design requirements is expressed by the weights. Methods for assessing weights are available in the field of decision analysis (Hwang and Yoon, 1981). A unique feature of the compromise DSP is that multiple design requirements can be considered at different priority levels using the preemptive deviation function. By exercising different scenarios, it is observed that the change of the deviation function has a significant impact on the design solution as well as the achievable DPIs. The results illustrate the conflicts between the requirements on volume and normal stress and the best achievable performance levels for each.

Impact of the preference function

We modify the preference function for normal stress to study the impact of the preference function on the design solutions. In Figure 8, the achieved normal stress is compared for two design scenarios under which different preference functions are used. Scenario (a), which is indicated by the dashed curves, represents the situation in our initial design model with the "smaller-the-better" (STB) type preference function. In Scenario (b), the preference function is changed to the "center the better" (CTB) as depicted by the solid lines. It is noted that with the CTB criterion, the mean of the normal stress distribution shifts to the right. This is reasonable since the maximum overlap between the performance distribution and the preference function would be achieved with this shift.

Insert Figure 8 Performance Variations for Different Preference Functions

3.4 Verification Studies

To illustrate the benefits of developing flexible solutions versus choosing a single point solution, we compare the fluctuations of design performance over the range of flexible solutions from our probabilistic-based model (simulation I) to those fluctuations over the same range size but using the singe point solution from the conventional model as the center (simulation II). Specifically, according to the results in Table 1, in simulation I, we use x_1 [42.2484 ± 4.1985] mm, x_2 [525.865 ± 56.767] mm as the range; in simulation II, we use x_1 [40.5785 ± 4.1985] mm, x_2 [547.989 ± 56.767] mm. In each case, 3000 points are randomly picked across the design space formed by the ranges.

It is noted that all the test points over the range of solutions from the probabilistic-based model are within the feasible design space (constraint = 0). This verifies that the ranged set of solutions can fully satisfy the constraint. However, when the same size of range varies around the single point solution obtained from the conventional model (simulation II), only a part of the test points (2506 of 3000) fall within the feasible design space. We can thus conclude that the range of solutions using our proposed probabilistic-based model is more flexible and reliable if the design condition changes.

To verify the normality assumption for the deviations of performance, a comparison of the deviations of performance obtained from the 3000 simulations and from the estimations using the normal distribution based on the statistical data (μ and σ) is conducted. It is noted that, for both volume and normal stress, the assumption is valid and the actual fluctuation of design performance across the range of solutions falls within the predicted range [μ-3σ, μ+3σ].

4. CONCLUSION

A probabilistic-based design model is developed in this paper as a basis for achieving flexibility in a design process. The flexibility is obtained by developing a range of design solutions instead of a point solution. The flexibility is also provided to designers in specifying a ranged set of design requirements with varying degree of desirability. There are several major advantages of using our approach:

- The interval representation of design variables allows designers to look for a range of “satisficing’, or equivalent good solutions. This is superior to the conventional approach in which only a single solution is sought. The uncertainties associated with specifying the desired performance levels for multiple design requirements are captured by using the preference functions to consider a ranged set of design requirements with varying degree of desirability.
- The Design Preference Index, DPI, is a unitless measure of the goodness of a set of design solutions in successfully satisfying a ranged set of design requirements. Using the compromise DSP, the tradeoffs between multiple design requirements could be achieved by separating the objectives of maximizing the DPIs as close as

possible to 1 as individuals goals. Regardless of the differences in physical units, tradeoffs between multiple design requirements are made by setting the weights of relative importance or the priority levels in the deviation functions.

- Our proposed model is a generic design model since it could be used for developing a range of solutions in situations when the desired degree of flexibility is different. The desired flexibility of design variables is set by the minimum range of design variables in our example. When α is equal to 0, the problem is equivalent to finding a single point solution in conventional designs.
- We utilize the statistical approach to the evaluations of design performance distribution over a range of solutions. Our verification studies illustrate the effectiveness of using the normal distribution assumption. The approach is especially useful for design problems where there are no explicit functional relationships and the system performance is computationally expensive to evaluate.

We anticipate that the probabilistic-based design model can be used in various stages of product development to provide flexibility in design. When design is in its later stages, the minimum range of solutions should be reduced since the design freedom usually decreases when the design proceeds. Future investigation is anticipated as for how to improve the computational efficiency in evaluating the performance variation when the system performance is computationally expensive but not necessarily normal. We are also interested in searching for solution sets that are not necessary formed by the ranges of individual variables but would assume irregular shape in a solution space.

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APPENDIX

Insert Figure A.1 The Two-Bar Structure

Insert Figure A.2 The Conventional Compromise DSP for a Two-Bar Design

List of Figures:

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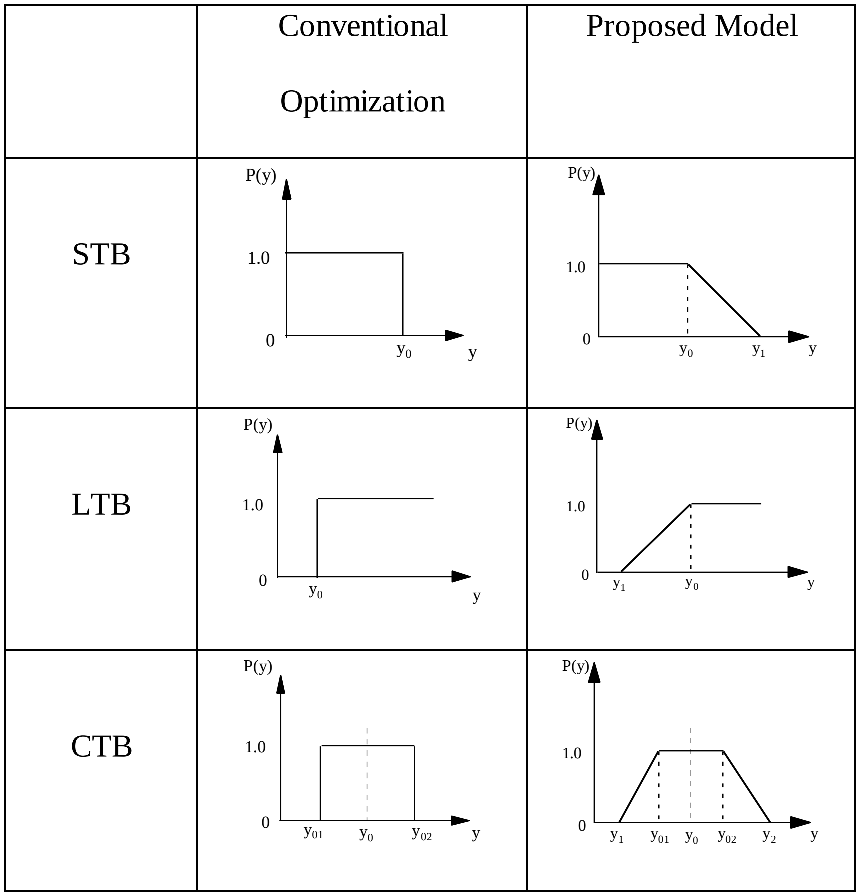


Figure 1 Preference Functions in the Proposed Model

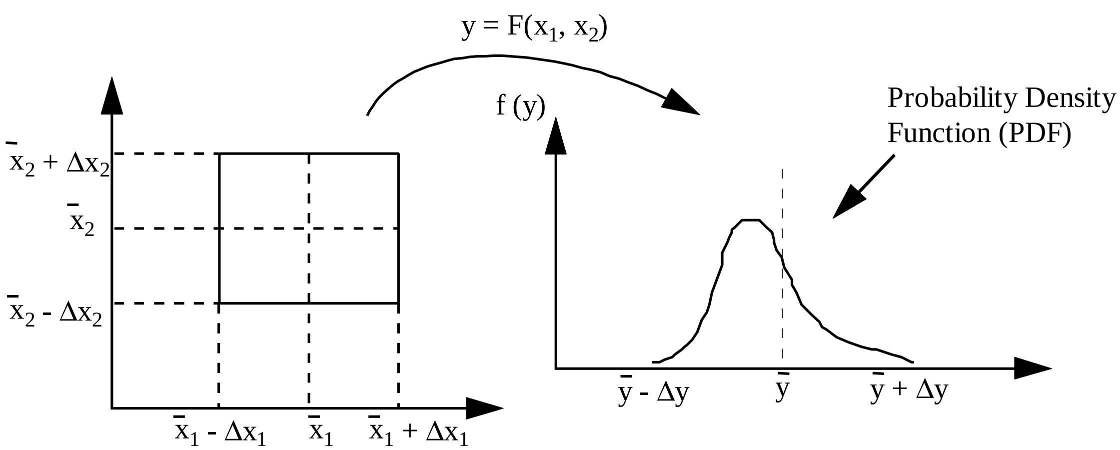


Figure 2 Mapping between Design Variables and Design Performance

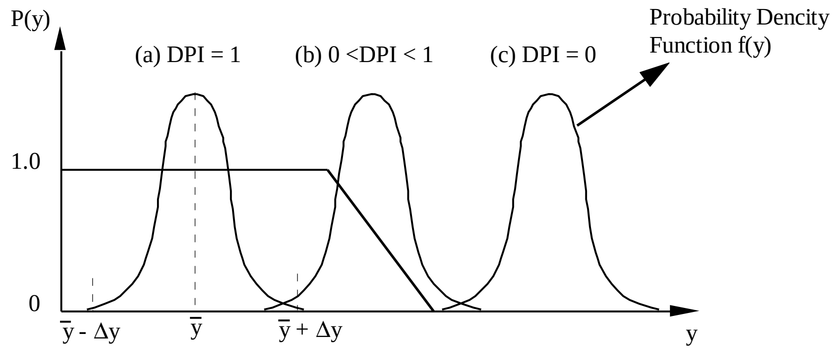


Figure 3 Design Preference Index

Given

An alternative to be improved. Assumptions used to model the domain of interest.

The system parameters:

n	number of system variables	q	inequality constraints
p + q	number of system constraints	m	number of system goals
$g_i(\mathbf{x})$	system constraint function		
$Z_k(d_i)$	function of deviation variables to be minimized at priority level k		

Find

$x_i \quad i = 1, \ldots, n$

$d_i^-, d_i^+ \quad i = 1, \ldots, m$

Satisfy

System constraints (linear, nonlinear)

$g_i(\mathbf{x}) = 0 ; \quad i = 1, \ldots, p$

$g_i(\mathbf{x}) \geq 0 ; \quad i = p+1, \ldots, p+q$

System goals (linear, nonlinear)

$A_i(\mathbf{x}) + d_i^- - d_i^+ = G_i ; \; i = 1, \ldots, m$

Bounds

$x_i^L \leq x_i \leq x_i^U ; \; i = 1, \ldots, n$

$d_i^-, d_i^+ \geq 0 ; \quad i = 1, \ldots, m; \quad d_i^+ \cdot d_i^- = 0; \quad i = 1, \ldots, m$

Minimize

Preemptive deviation function (lexicographic minimum)

$\mathbf{Z} = [\, Z_1(d_i^-, d_i^+), \ldots, Z_k(d_i^-, d_i^+) \,]$

Figure 4 Mathematical Form of a Conventional Compromise DSP

Given The system parameters:

$n, q, m, \Delta y_{imax}, g_i(\mathbf{x}), F_i(\mathbf{x}), P_i(y_i), f_i(y_i), Z_k(d_i)$

Find

System variables

$\bar{x}_i, \Delta x_i \quad i = 1, \ldots, n; \quad d_i^-, d_i^+ \quad i = 1, \ldots, m$

Satisfy

System constraints

$$g_i^*(\bar{x}, \Delta \mathbf{x}) \geq 0 ; \quad i = 1, \ldots, q \qquad [4]$$

$$\Delta x_i \geq \Delta x_{imin} ; \quad i = 1, \ldots, n \qquad [5]$$

$$\Delta y_i \leq \Delta y_{imax} ; \quad i = 1, \ldots, m \qquad [6]$$

System goals

$$DPI_i + d_i^- - d_i^+ = 1; \quad i = 1, \ldots, m \qquad [7]$$

$$DPI_i = \int_{\bar{y}_i - \Delta y_i}^{\bar{y}_i + \Delta y_i} P_i(y_i) f_i(y_i) dy_i \qquad [8]$$

$\bar{y}_i$ is the mean of design performance.

Bounds

$$\bar{x}^L \leq \bar{x} \leq \bar{x}^U ; \quad i = 1, \ldots, n \qquad [9]$$

$$\Delta x_i^L \leq \Delta x_i \leq \Delta x_i^U ; \quad i = 1, \ldots, n \qquad [10]$$

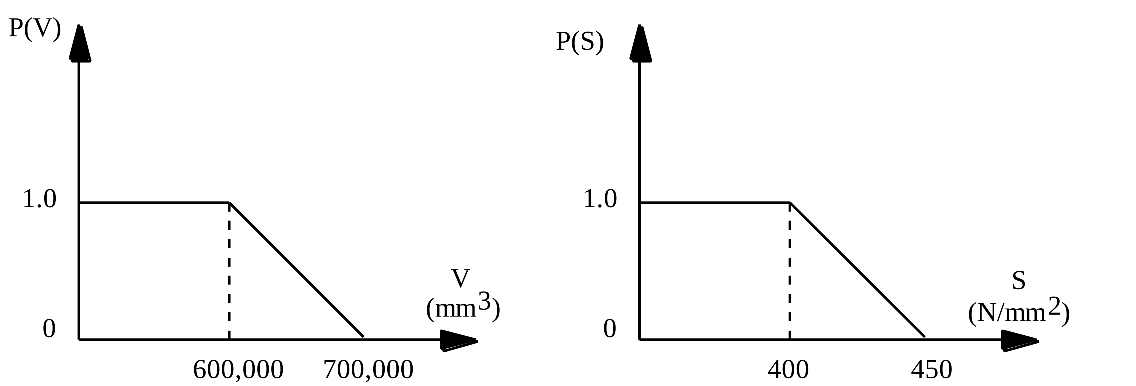
$$d_i^-, d_i^+ \geq 0 ; \quad i = 1, \ldots, m \qquad [11]$$

$$d_i^+ \cdot d_i^- = 0; \quad i = 1, \ldots, m \qquad [12]$$

Minimize

$$Z = [Z_1(d_i^-, d_i^+), \ldots, Z_k(d_i^-, d_i^+)] \qquad [13]$$

Figure 5 The Probabilistic-Based Multiobjective Design Model



(a) The preference function of the volume (b) The preference function of the normal stress

Figure 6 Preference Functions for Volume and Stress

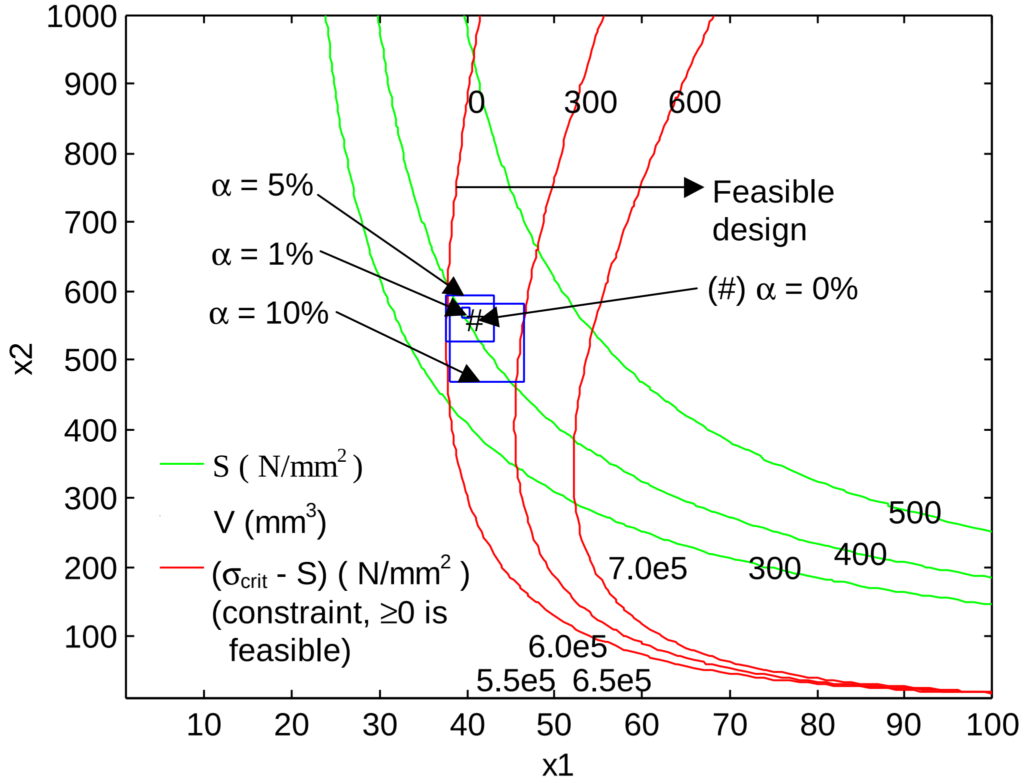


Figure 7 Results for Illustrating the influence of a

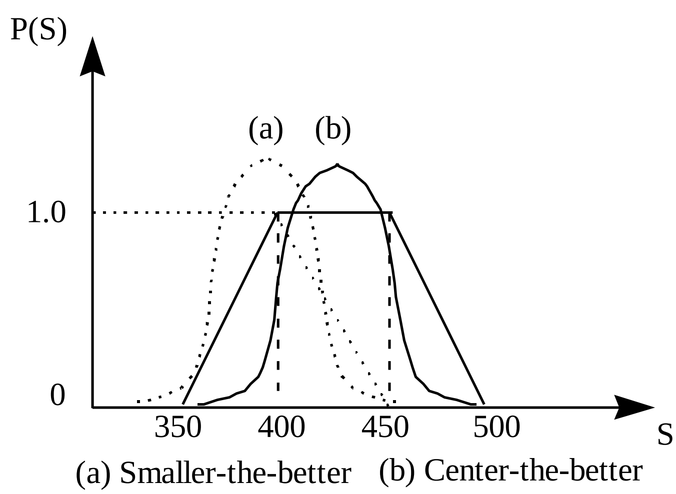


Figure 8 Performance Variations for Different Preference Functions

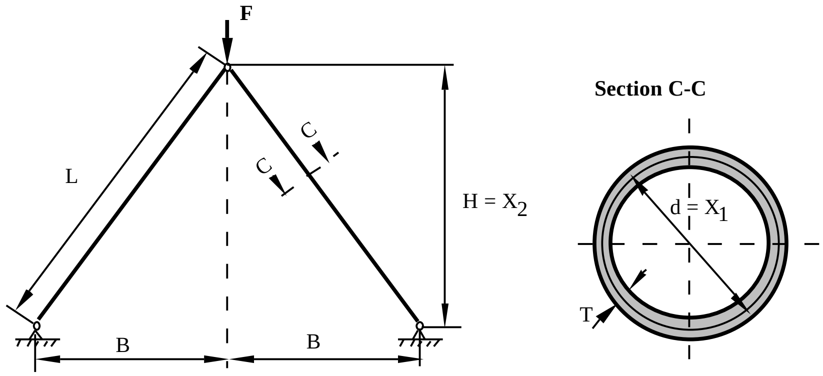


Figure A.1 The Two-Bar Structure

Given	External Force	F	= 150 KN
	Thickness of the cross section	T	= 2.5 mm
	Width of the structure	B	= 750 mm
	Elastic Modulus	E	= 210,000 N/mm^2

$$V(x_1, x_2) = 2\,L\,A = 2\pi T x_1 \sqrt{B^2 + x_2^2}$$

$$S(x_1, x_2) = \frac{F}{2\pi T x_1 x_2} \sqrt{B^2 + x_2^2}$$

$$\sigma_{crit} = \frac{F_{crit}}{A} = \frac{1}{A} \bullet \frac{\pi^2 EI}{L^2} = \frac{1}{8} \bullet \pi^2 E \frac{T^2 + x_1^2}{B^2 + x_2^2}$$

Find x_1, x_2

Subject to

Constraints $S(x_1, x_2) = \sigma_{crit}(x_1, x_2)$

Goals $600{,}000 / V(x_1, x_2) + d_1^- - d_1^+ = 1$

$400 / S(x_1, x_2) + d_2^- - d_2^+ = 1$

Bounds $1 = x_1 = 100$

$10 = x_2 = 1000$

Minimize $Z = [\, Z_1(d_1^+), Z_2(d_2^+) \,]$

Figure A.2 The Conventional Compromise DSP for a Two-Bar Design

Table 1 Results for the Two-Bar Problem

		Probabilistic Decision Model	Conventional Model
Design Solution	Diameter x_1(mm)	42.2484 ± 4.19848	40.5785
	Height x_2 (mm)	525.865 ± 56.7667	547.989
Performance	Volume V(mm^3)	608013 ± 83372.5	592063
	Normal stress S (N/mm^2)	394.416 ± 63.1151	389.894
DPI	DPI_V	0.84487	--
	DPI_S	0.882679	--
Goal Deviations	$[d_1^-, d_1^+]$	[0.155513, 0]	[0, 0.01341]
	$[d_2^-, d_2^+]$	[0.117321,0]	[0,0.00277]
	$0.5d_1^- + 0.5\ d_2^-$	0.136417	0