Satisfying Ranged Sets of Design Requirements using Design Capability Indices as Metrics

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Abstract

In this paper we propose the use of design capability indices for use in robust design. These indices allow the design requirements to vary within a certain *range*. Design capability indices are based on process capability indices from statistical process control and provide an alternate approach to the use of Taguchi's signal-to-noise ratio which is often used for robust design. Successful implementation of design capability indices ensures that a range of design solutions, representing a family of designs, conforms to a given ranged set of design requirements. The essence of using a design capability index is to predict the spread of the design performance so that the capability of a *family* of designs to satisfy a range of design requirements can be assessed. The design of a solar powered irrigation system is presented to demonstrate the usefulness of design capability indices.

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NOMENCLATURE

$A_j(x)$	Goals in the compromise DSP
$C_i(x)$	Constraints in the compromise DSP
C_d, C_{dk}, C_{dl}, C_{du}	Design capability indices extended from process capability indices (C_p, C_{pk}, C_{pl}, C_{pu}) for Statistical Process Control
d_i^+, d_i^-	Deviation variables in the compromise DSP
DSP	Decision Support Problem
LRL	Lower Requirement Limit
LSL	Lower Specification Limit
SPC	Statistical Process Control
URL	Upper Requirement Limit
USL	Upper Specification Limit
X_A	Cycle maximum pressure (MPA)
X_B	Cycle maximum temperature (K)
X_C	Collector maximum temperature (K)
X_D	Working fluid flow rate (kg/s)
Z	Deviation function in the compromise DSP
$\mathbf{X}$	Control factors
y	Response
$\hat{y}$	Estimated response
μ, $\bar{\bar{x}}$	Mean values: general notation, general notation for use in SPC.
σ	Statistical standard deviation
? x_i	the deviation of a sensitive design variable (control parameter)

1. OUR FRAME OF REFERENCE

An increasing number of companies are striving to deliver greater quality, enhanced customization, faster response times and more innovative designs at lower prices (cf., Bower and Hout, 1988; Stalk and Hout, 1990). This requires flexible designs that are readily adaptable to changing customer requirements. Flexibility is particularly important in the early stages of design when little is known about a design and its requirements; changes in customer requirements are almost inevitable in the later stages of design. *How many times do we hear about changes in customer requirements in the later design stages forcing a project to be scraped or costs to skyrocket?* To minimize the effects of these changes, designers must be able to either, (a) anticipate changes that may occur during the product development process or (b) provide additional flexibility, allowing designs to be readily adapted to changing customer requirements. Our focus in this paper is on providing additional flexibility.

In this paper, we address two specific issues related to flexibility, namely, flexibility in the design requirements and flexibility in the design variables (top-level design specifications[1]) themselves. This is further explained as follows.

- Early in the design timeline, some of the design requirements for a system may be uncertain. A designer may have some knowledge of the *range* of these design requirements, but may not be clear as to the exact targets that should be met. Therefore, in order to maintain flexibility in the later design stages, a designer may allow the design requirements to vary over a range rather than

[1] Top-level design specifications are those design variables used to describe the system in the early stages of design.

set points targets. The question thus becomes, *how does one ensure that these ranged design requirements are satisfied by a design?*

- Instead of looking for a single solution for a design problem, a designer might be interested in finding a range of values for each design variable in order to enhance system flexibility. In this case, it is necessary to determine whether the system performance of the family of designs satisfies the given design requirements.

1.1 Methods for Modeling Design Variations

Modeling the variations of design parameters (or variables) and the corresponding fluctuations of design performance has been studied extensively. The methods employed in the literature are generally classified into four categories: fuzzy set theory, interval methods, sensitivity analysis methods, and probabilistic-based methods such as robust design. Wood and Antonsson (1987) employ fuzzy set theory to handle imprecision in preliminary design. In later work, they compare fuzzy set theory and probabilistic methods for dealing with different types of uncertainties (Wood and Antonsson, 1990). Chen and Ward (1995) develop a quantitative inference method based on interval arithmetic for problems with sets of design solutions. Using their method, each design parameter is specified as a range or an interval. A limitation of their method is that the problems must be formulated by linear relationships between the design behavior and design variables. In complex design problems, such a condition could seldom be satisfied. Sandgren et al. (1985) utilize sensitivity analysis to minimize the sensitivity of a design to variations in uncontrollable parameters. They seek “a set of design variables which produces the smallest change in the constraint function for a unit change in the uncontrollable parameters.” Using their approach, variation in uncontrollable parameters is modeled as an interval, usually taken as a certain percentage of the nominal value.

1.2 The Improved Robust Design Methods

In the category of probabilistic-based methods, the robust design method is perhaps the most well-known approach for handling variation in the design process. While sensitivity analysis is focused on reducing the local rate of change of design performance, the robust design method or Taguchi method (Phadke, 1989; Taguchi, 1993) goes one step further by introducing uncertainty or noise in the system and generating optimal solutions that could reduce the impact of the uncertainty in a global scale. This concept has been extended into the early stages of design to improve the "quality" of design decisions in addition to improving the quality of a product as originally introduced by Taguchi. As an example, Chang and Ward (1994) present a procedure based on Taguchi's method to reason about sets of design alternatives which are conceptually robust, i.e., insensitive to conceptual noise--variations in the design posed by other members of the design team or physical noise. Although the robust design technique has been widely accepted as a method for designing quality into products and processes, there are certain mathematical limitations (cf., Chen et al. 1996) associated with the methods broadly classified as Taguchi methods. Alternative approaches have been proposed by the researchers in the engineering design community. Parkinson, et al. (1993) present a general approach for robust optimal design, by which tolerances are incorporated on all model variables and parameters; worst case scenario and statistical analyses are introduced to address the issue of design feasibility under robust design considerations. Balling, et al. (1986) present two formulations of manufacturing tolerance problems in design optimization. In both cases, an optimal design without tolerance considerations is first found, then a new design is sought to either minimize the distance between the new design and the initial optimal design when tolerances are given, or maximize the tolerance range on the optimal design when tolerances are expected to be determined. In the work of Sundaresan, et al. (1993), a sensitivity index (the root mean square of the deviation caused by the worst case variation in the design variables and design parameters) is used as a measurement for robustness. Otto and Antosson (1993) discuss several extensions to the Taguchi method to incorporate necessity requirements (i.e., an interval of values all of which a design must satisfy) and possibilistic uncertainty (parameters which are free to change within a design over which the designer has no control).

We have introduced our own variation to the Taguchi method to solve two broad categories of robust design problems (see Chen, et al., 1996), namely, (1) robust design associated with the minimization of variations in performance caused by variations in noise factors (uncontrollable parameters), and (2) robust design associated with the minimization of variations in performance caused by variations in control factor (design variables). The concept is extended for making reliable design decisions during the design process when subject to uncontrollable parameters (Type I) or design variables which vary within a range (Type II). Using our proposed procedure (see Chen, et al., 1996), a robust design problem is modeled using a compromise Decision Support Problem (DSP) with two separate objectives for "bringing the mean on target" and "minimizing the deviation" of the system performance. The compromise DSP is a multiobjective mathematical construct which is a hybrid formulation based on mathematical programming and goal programming (Mistree, et al., 1993), see Figure 1. In the compromise DSP, each goal has two associated deviation variables d_i^- and d_i^+ which indicate the extent of the deviation from the target G_i. Goals may either be weighted in an Archimedean solution scheme or, using a preemptive approach, rank-ordered into priority levels to effect a solution on the basis of preference (Mistree, et al., 1993). The goals are transformed into minimizing the total deviation function, a formula comprised of deviations variables, as indicated by Z in Figure 1. For the preemptive approach, we use the lexicographic minimum concept (Ignizio, 1985) to quickly evaluate different design scenarios by changing the priority levels of the goals to be achieved. The compromise DSP is solved using the ALP algorithm (Mistree, et al., 1993), a part of DSIDES (Decision Support in Designing Engineering Systems, (Reddy, et al. 1992)).

Given

An alternative to be improved. Assumptions used to model the domain of interest.

The system parameters:

n	number of system variables	q	inequality constraints
p + q	number of system constraints	m	number of system goals
$g_i(\mathbf{X})$	system constraint function:		
$f_k(d_i)$	function of deviation variables to be minimized at priority level k for the preemptive case.		

Find

X_i $\quad i = 1, \ldots, n$

d_i^-, d_i^+ $\quad i = 1, \ldots, m$

Satisfy

System constraints (linear, nonlinear)

$g_i(\mathbf{X}) = 0$; $\quad i = 1, \ldots, p$

$g_i(\mathbf{X}) = 0$; $\quad i = p+1, \ldots, p+q$

System goals (linear, nonlinear)

$A_i(\mathbf{X}) + d_i^- - d_i^+ = G_i$; $i = 1, \ldots, m$

Bounds

$X_i^{min} = X_i = X_i^{max}$; $\quad i = 1, \ldots, n$

$d_i^-, d_i^+ = 0$; $\quad i = 1, \ldots, m$

$d_i^- \cdot d_i^+ = 0$; $\quad i = 1, \ldots, m$

Minimize

Preemptive deviation function (lexicographic minimum)

$\mathbf{Z} = [\, f_1(d_i^-, d_i^+), \ldots, f_k(d_i^-, d_i^+) \,]$

Figure 1 - Mathematical Form of a Compromise DSP

While the concept of robust design has been successfully extended to making reliable decisions in the early stages of design, we find it is not always effective to model the two aspects of robust design as separate goals. An example is the case in which *satisfying a range of design requirements is the major concern*, see Figure 2.

In Figure 2, the quality distributions of two different designs (I and II) are illustrated. Both designs have the same mean value but different deviations. If the two aspects of robust design are modeled as separate goals, the design with the least deviation (Design I) is chosen because both designs have the same performance mean. However, in this particular situation where the mean of the design performance lies outside the range of requirements, a smaller fraction of the performance falls inside the upper and lower requirement limits (URL and LRL, respectively) with a thinner bell shape, i.e., the shadowed area which is enclosed by A, B, and C (Design I) is smaller than the area enclosed by A', B' and C' (Design II). This indicates that Design II is preferred to Design I when satisfying a "ranged set of requirements" is the major concern. An alternate evaluation criterion is thus required to replace the approach of modeling the two aspects of robust design as separate goals. We recommend using design capability indices which are useful for meeting a range of design requirements.

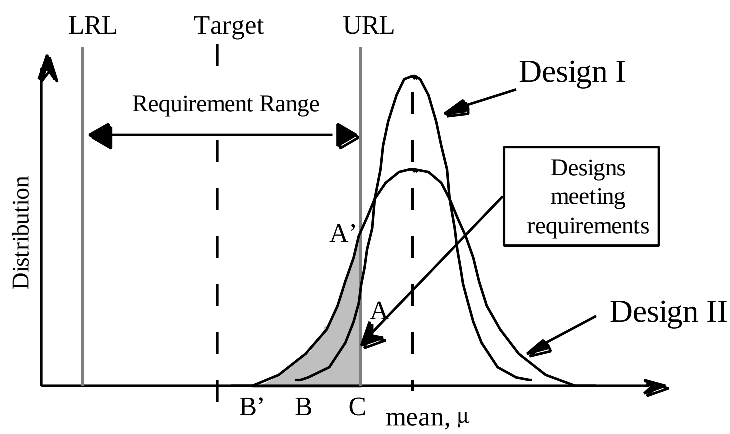

Figure 2 - Quality Performance Distribution of Two Designs

2. DEVELOPMENT OF DESIGN CAPABILITY INDICES

Design capability indices are based on process capability indices from Statistical Process Control (SPC), an approach for quality control rooted in manufacturing. As outlined in (Kalpakjian, 1991), the SPC technique uses a combination of control charts, control limits, and process capability indices to monitor and control manufacturing processes. Capability indices have been developed to compare the distribution of a process to the specification limits.

2.1 Process Capability Indices

Process capability is defined as *the limits within which individual measurement values resulting from a particular manufacturing process would normally be expected to fall* (Kalpakjian, 1991). It is used to determine whether a process is capable of meeting established customer/designer specifications (Brassard and Ritter, 1994). A stable process is represented by a measure of its variation; six standard deviations ($\pm 3\sigma$) is commonly chosen. The simplest measure of process capability, C_p, compares the variation of a process to the customer specifications through the equation

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} \qquad (1)$$

where USL is the upper specification limit, LSL is the lower specification limit, and $\hat{\sigma}$ is the standard deviation of the process. The relationship between C_p and the user specification limits is illustrated in Figure 3. As shown in the figure, if C_p is less than one, the process variation exceeds the specification limits, and defectives are being produced. Conversely, if C_p is greater than one, the process variation is less than the specification limits, and no defectives are being made (provided the process mean is centered on the target value). If C_p is equal to one, the process is just meeting customer specifications, and as shown in Figure 3, a minimum of 0.3% defectives will be made (again, provided that the process mean is centered on the target value).

Bringing C_p as close as possible to 1 using Eqn. (1) is often referred to as a three-sigma approach in manufacturing and relates to an expected defect rate usually expressed in parts per million. If a lower percentage of defectives is desired, a higher target value needs to be assigned for maximizing C_p. The six-sigma approach is becoming more and more popular in industrial quality programs (McFadden, 1993). Using the six-sigma approach, $C_p = 2$ is the design target based on Eqn. (1), and the defect rate can be reduced to less than 3.4 per million.

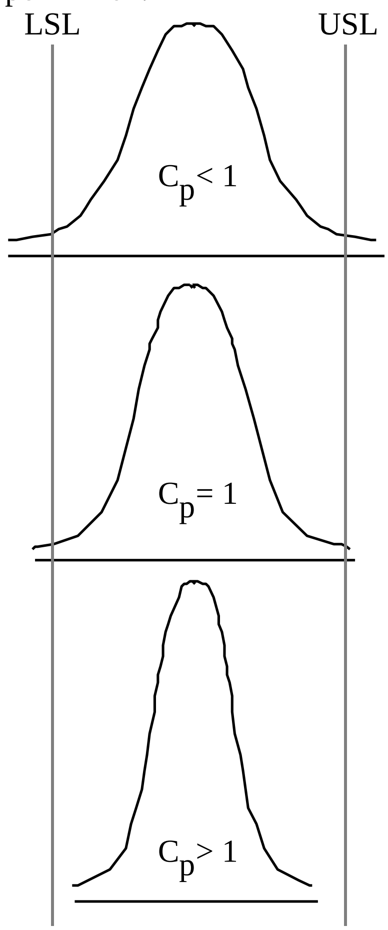

Figure 3 – Relationship Between Process Capability Index and Customer Specifications (Brassard and Ritter, 1994)

Although C_p is a measurement of the spread of the process in relation to specification width, it *does not measure how well the process average is centered about the target value*. The process capability indices C_{pl}, C_{pu}, and C_{pk} are used to measure both process variation with respect to customer specifications and the location of the process average. C_{pl} and C_{pu} are process capability indices for single-sided specification limits while C_{pk}

measures process capability for two-sided specification limits. C_{pk} is commonly used to measure the process capability when the process mean is off the target value, see Figure 4, and is taken as the smaller of C_{pl} and C_{pu}. C_{pl}, C_{pu}, and C_{pk} are calculated using the equation:

$$C_{pl} = \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}} \; ; \; C_{pu} = \frac{USL - \bar{\bar{x}}}{3\hat{\sigma}} \; ; \; C_{pk} = \min\{C_{pl}, C_{pu}\} \qquad (2)$$

where $\bar{\bar{x}}$ is the process mean, $\hat{\sigma}$ is the standard deviation of the process, and USL and LSL are the upper and lower customer specification limits, respectively. For a process with its mean centered on the target value, C_{pk} and C_p are equal.

In Figure 4, the process mean, $\bar{\bar{x}}$, is greater than the target value, and C_{pk} is equal to C_{pu} since it is the smaller of C_{pl} and C_{pu}. This is a good example where C_p alone is not enough. Note that the process is “capable” when it is centered around the target ($C_p > 1$, since the variation is less than the specification width, refer to Figure 3). However, because the process is not centered about the target, defectives are being made, since part of the process curve falls outside of the upper specification limit, USL. In this case, it is preferable to monitor C_{pu}. Following the guidelines given previously in Figure 3 for C_p, this process would be “in-control” if C_{pu} is greater than or equal to 1; however, in Figure 4, C_{pu} is less than 1 since the process is not centered and the process is not in-control

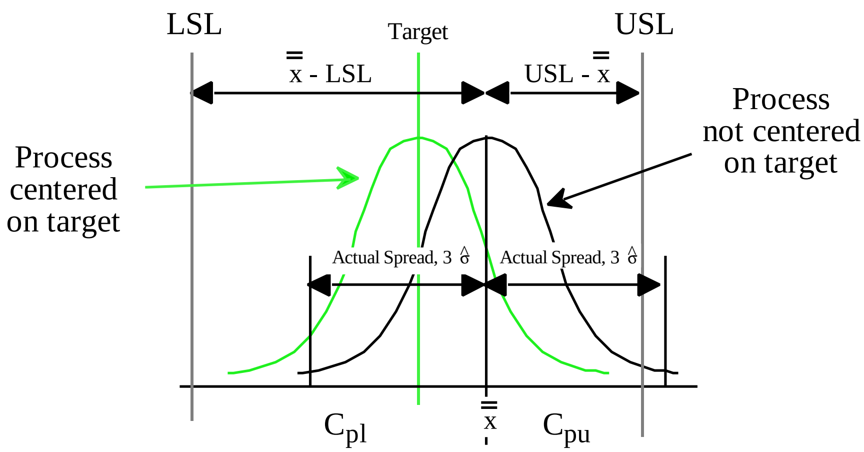

Figure 4 – Process Capability Indices for Process Mean not Centered on Target

(Brassard and Ritter 1994)

2.2 The Use of Design Capability Indices in Robust Design

Since the measurements for performance variations caused by manufacturing variations and performance variations corresponding to a range of solutions in a design system are similar, we extend the concept of measuring process capability into design to measure the capability of a design to satisfy its requirements. Specifically, we propose to use design capability indices to determine whether a range of design solutions, representing a family of designs, is capable of satisfying a ranged set of design requirements. Here we first assume that the system performance of a family of designs is approximated by a distribution with a mean, μ, and a standard deviation, σ. The statistical representation is particularly useful when techniques such as design of experiments (Box et al, 1978) are used to evaluate the performance distribution for complex problems, in which only closed-form (or black box) types of programs are available to simulate the design performance. In Figure 5, a normal distribution is given as an example. The process capability indices, C_{pl}, C_{pu}, and C_{pk}, become design capability indices C_{dl}, C_{du}, and C_{dk} which *measure the portion of the family of designs that satisfies the ranged design requirement*. In the figure, the upper and lower specification limits (USL and LSL, respectively) used with C_{pk} are replaced by upper and lower requirement limits (URL and LRL, respectively) since we are concerned with satisfying design requirements and not machining specifications. Consequently, C_d, which is based on C_p and does not account for the mean being off target, is not used as a design capability index; *only* C_{dl}, C_{du}, and C_{dk}, are used for different design situations, see Figure 5.

As illustrated in Figure 5, for the case in which nominal is better, i.e., a requirement is given with some acceptable lower and upper limits, finding a family of designs with C_{dk} =

1 will satisfy the design requirements while a family of designs with $C_{dk} < 1$ has some portion which falls outside of the requirement limits. In this scenario, C_{dk} is computed using Eqn. (3), and C_{dk} is taken as the minimum of C_{dl} and C_{du}.

$$C_{dl} = \frac{\mu - LRL}{3\hat{\sigma}}\ ; C_{du} = \frac{URL - \mu}{3\hat{\sigma}}\ ; C_{dk} = \min\{C_{dl},\ C_{du}\}. \qquad (3)$$

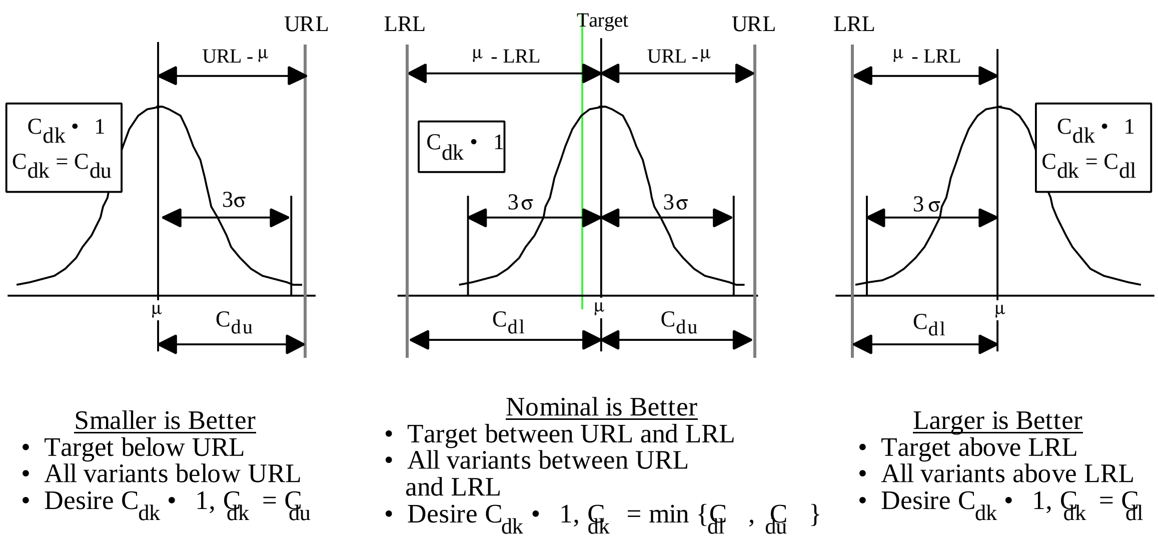

Figure 5 – Design Capability Indices

For the case where smaller is better (the deflection of a beam should be less than 0.5 mm for example), designs with a $C_{dk} = 1$ are capable of satisfying the requirement, where C_{dk} is equal to C_{du}. Designs with a $C_{du} < 1$ do not meet this requirement (refer to Figure 4) since a large portion of the distribution falls outside of the upper requirement limit. Similarly, for the case where larger is better (the target efficiency of a family of engines should be 30% or better, say), designs with a $C_{dk} = 1$ are capable of meeting this requirement, and C_{dk} is equal to C_{dl} as shown in Figure 5. Distributions with a $C_{dl} < 1$ will have designs which do not meet this requirement.

When the statistical analysis is used to evaluate the distribution of performance, the number of standard deviations, k, used in the calculation indicate the percentage of this distribution conforming to the design requirements. Examples of the relation of k to the percentage are listed in Table 1. For instance, in Eqn. (3), since three standard deviation (3σ) is used, k is equal to 3. From the data in Table 1, it implies that when C_{dk} reaches 1, 99.865% of the performance distribution conform to requirements assuming that the performance is normally distributed. When the system function is complex, it is difficult to perform the judicious evaluation to determine performance distribution; however, distributions of design performance will tend towards normality from the Central Limit Theorem. It is observed that three standard deviation (3σ) is usually sufficient to capture the total deviation of performance when simulation-based analysis (such as Monto-Carlo simulation and design of experiments) is employed to measure the variations of performance. These observations are further demonstrated in our example problem. In the statistical community, modifications to the process capability indices for different variances have been proposed (Johnson, et al., 1992; NG and Tsui, 1992; Rodriguez, 1992). Design capability indices could be modified similarly.

Table 1 – The Relation of k to the Percentage of Performance Conforming to Requirements

k (number of standard deviations)	**1**	**2**	**3**	**4**
% of performance conforming to requirements	84.13	97.725	99.865	99.9968

Note: k is different from C_{dk}

The concept introduced so far is further extended to the evaluation of design capability when the deviation of design performance cannot be quantified by a statistical distribution.

If we assume that the design variable deviates ±? x around its nominal value x_0, then the corresponding deviation of system performance is ±? y around its mean value, μ, which can be calculated by:

$$\mu = f\,(x_0) \tag{4}$$

The following equation based on the first order Taylor expansion can then be used to estimate ? y when deviation ? x is small:

$$\Delta y^2 = \sum_{i=1}^{k} \left(\frac{\partial f}{\partial x_i}\right)^2 \Delta x_i^{\,2} \tag{5}$$

The representation of the performance deviation in Eqn. (5) is close to the worst case scenario which assumes that all fluctuations may occur simultaneously in the worst possible combination. Correspondingly, the system performance varies between μ - ? y and μ + ? y, and Eqn. (3) for defining the design capability indices is modified as:

$$C_{dl} = \frac{\mu \text{ - LRL}}{\Delta y}\,;\; C_{du} = \frac{\text{URL - }\mu}{\Delta y}\,;\; C_{dk} = \min\,\{C_{dl},\, C_{du}\}\,. \tag{6}$$

Using this concept, a family of designs based on a *ranged set* of top-level design specifications can be designed in an efficient and effective manner. By measuring the mean of the performance and the corresponding variance, the top-level design specifications for a system can be determined which satisfy the range (or point) design requirements.

Having introduced design capability indices, the compromise DSP in Figure 1 is particularized in Figure 6. In this formulation, the design capability indices can be used for either constraints or goals in robust design, depending on whether satisfying a range of design requirements is either a wish or a demand.[3] If a requirement is a wish, then bringing C_{dk} as close to 1 as possible can be used as a goal in the compromise DSP, Eqn. (7). When a requirement is a demand, then $C_{dk} = 1$ becomes a constraint in the compromise DSP. Therefore, instead of using a worst case scenario for constraints, satisfying constraints can be achieved by forcing the associated design capability indices to be greater than 1, Eqn. (8).

Given

Functions **y** including those ranged design requirements which are constraints, $g_i(\mathbf{X})$, and those which are objectives, $A_j(\mathbf{X})$

Deviations of the control variables, $\sigma_{\mathbf{X}}$ or ? x

Target *ranges* for the design requirements, URL_i and LRL_i

Find

The location of the mean of the control variables $\mu_{\mathbf{X}}$

Satisfy

Constraints: $C_{dk\text{-constraints}} = 1.$ (7)

Goals: $C_{dk\text{-objectives}} + d_i^- - d_i^+ = 1$ (8)

Bounds:

$d_i^-, d_i^+ = 0;$ $i = 1, \ldots, m$

$d_i^- \cdot d_i^+ = 0;$ $i = 1, \ldots, m$

Minimize

Deviation Function $Z=[f_1(d_i^-, \ldots, d_i^+) \ldots f_k(d_i^-, \ldots, d_i^+)]$

Figure 6 – The Compromise DSP Particularized for Designing Flexible Top-Level Design Specifications Using Design Capability Indices

The compromise DSP particularized in Figure 6 is used to find the mean location of design variables, $\mu_{\mathbf{X}}$, which could satisfy the range of design requirements either as a wish (Eqn. (7)) or a demand (Eqn. (8)). The range of design variables are represented by $\sigma_{\mathbf{X}}$ or ? x. Depends on the actual situation, the range could be given information or design variables

[3] Demands are requirements that must be met under all circumstances; wishes are requirements that should be taken into consideration whenever possible.

whose values need to be identified. Note that when a deviation function includes solely design capability indices, the negative deviation variable, d_i^-, is minimized and the target value is set to 1. *This ensures that the design capability index will be as close to one (1) as possible and the design distribution will be within the corresponding range for the design requirements*. From Eqn. (6), it is noted that, to some extent, maximizing C_{dk} is equivalent to minimizing the total performance deviations (? y) as well as bringing the performance mean away from the boundary of the specification range. To demonstrate the use of design capability indices, the design of a solar powered irrigation system is presented in the next section.

3. DESIGN OF A SOLAR POWERED IRRIGATION SYSTEM USING DESIGN CAPABILITY INDICES

The design of a solar powered irrigation system has been used previously to demonstrate our general robust design procedure for solving problems related to the two major types of robust design (Chen, et al., 1995); this problem was originally developed by Bascaran, et al (1989). Using the same example problem, we employ *design capability indices* to develop a *ranged* set of top-level design specifications which satisfies the given ranged set of design requirements.

3.1 The Problem Statement

The layout of the solar powered irrigation system is shown in Figure 7. It is assumed that parabolic trough, N-S tracking is used for solar collection, and water is the working fluid. The to-be-determined top-level design specifications for the system are the following:

- maximum operating pressure, Rankine Cycle, X_A (MPA)
- maximum operating temperature, Rankine Cycle, X_B (K)
- maximum temperature drop in the solar collectors, X_C (K)

- working fluid flow rate, X_D (kg/s)

In order to maintain flexibility in the next stage of design development (e.g., component design of the turbine, pump, collector, etc.), we will develop a *ranged* set of top-level design specifications for X_A, X_B, X_C and X_D rather than point solutions through the use of the compromise DSP and design capability indices. We assume, after normalization, within the limits of these four top-level design specifications, i.e., X_A [-1, 1], X_B [-1, 1], X_C [-1, 1], and X_D [-1, 1], the range for each top-level design specification is ? X_A = ? X_B = ? X_C = ? X_D = ±0.2. The system performance should satisfy the following *ranged* design requirements:

- pumped load (power output) should be between 17 kW and 23 kW,
- overall efficiency should be 19% or better, and
- economic benefits should be $145,000 or better.

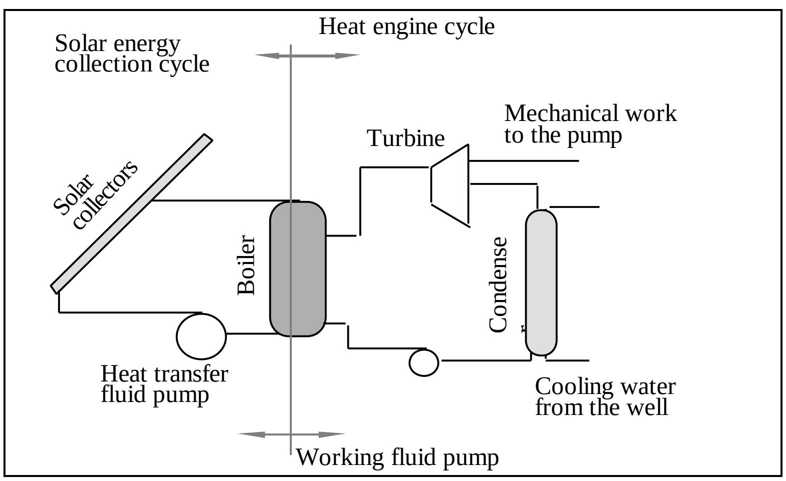

Figure 7 - Solar Powered Irrigation System

3.2 Formulating the Compromise DSP Incorporating the Design Capability Indices

The compromise DSP is formulated as shown in Figure 8, in which the design capability index, C_{dk}, is used to measure the portion of the family of designs (based on the *ranged* set of top-level design specifications) which satisfies the design requirements. As discussed in Section 2, in the situation when nominal is better, $C_{dk} = \min\{C_{dl}, C_{du}\}$; when smaller is better, $C_{dk} = C_{du}$; and when larger is better, $C_{dk} = C_{dl}$. In this problem, the design requirement for power is taken as a "nominal is best" case since there are both upper (URL) and lower limits (LRL) associated with the design requirement, i.e., power = [17-23 kW]. For efficiency and economic savings, a larger value is desired; therefore, there are only lower limits (LRL), and $C_{dk} = C_{dl}$ for these two design requirements. The LRL for efficiency is 19%, while the LRL for savings is set at $145,000.

Given

- Second order response surface models for Power, Overall Efficiency, and Savings as functions of X_A, X_B, X_C, X_D, i.e., POW(**X**), EFF(**X**), and SAV(**X**).
- ? X_A = ? X_B = ? X_C = ? X_D = 0.2 (normalized)
- The target ranges for the design requirements:
  - Power: 17 kW = T_{pow} = 23 kW (nominal is better)
  - Efficiency T_{eff} = 19% (larger is better)
  - Savings T_{sav} = $145,000 (larger is better)
- The targets for the design capability indices: C_{dk}-pow = C_{dk}-eff = C_{dk}-sav = 1.

Find

- The mean of the top-level design specifications
  - X_A, Normalized cycle maximum pressure n.m.u.
  - X_B, Normalized cycle maximum temperature n.m.u.
  - X_C, Normalized collector maximum temperature n.m.u.
  - X_D, Normalized working fluid flow rate n.m.u.
- The values of deviation variables associated with the goals d_i^-, d_i^+ n.m.u. (i =1, 3)

Satisfy

- The goals:

  Achieve power within the range of specification (nominal is better)

  $C_{dk\text{-}pow} + d_1^- - d_1^+ = 1,$ (9)

  $C_{dk\text{-}pow} = \min\{C_{dl\text{-}pow}, C_{du\text{-}pow}\}$ where

  $C_{dl\text{-}pow} = \{POW(\mathbf{X}) - 17\}/?\,POW(\mathbf{X})$, & (10a)

  $C_{du\text{-}pow} = \{23 - POW(\mathbf{X})\}/?\,POW(\mathbf{X})$ (10b)

  Achieve the efficiency within the range of specification (the larger is better)

  $C_{dk\text{-}eff} + d_2^- - d_2^+ = 1,$ (11)

  $C_{dk\text{-}eff} = C_{dl\text{-}eff} = \{EFF(\mathbf{X}) - 0.19\}/?\,EFF(\mathbf{X})$ (12)

  Achieve the savings within the range of specification (the larger is better)

  $C_{dk\text{-}sav} + d_3^- - d_3^+ = 1,$ (13)

  $C_{dk\text{-}sav} = C_{dl\text{-}sav} = \{SAV(\mathbf{X}) - 145{,}000\}/?\,SAV(\mathbf{X})$ (14)

- Bounds on the design variables

  -1 = X_A = 1

  -1 = X_B = 1

  -1 = X_C = 1

$-1 = X_D = 1$

- $d_i^+ \cdot d_i^- = 0$, with $d_i^+, d_i^- = 0$

Minimize

- The sum of the deviation variables associated with:
  - the design capability index for power, $C_{dk\text{-}pow}$, d_1^-
  - the design capability index for efficiency, $C_{dk\text{-}eff}$, d_2^-
  - the design capability index for savings, $C_{dk\text{-}sav}$, d_3^-

$$Z = [\ f_1(d_1^-),\ f_2(d_2^-),\ f_3(d_3^-)] \quad (15)$$

Figure 8 – The Compromise DSP for Designing Flexible Top-Level Design Specifications Using Design Capability Indices

The performance relationship is determined from an integrated thermodynamic property prediction simulator (Shamsundar, 1989) and an economic analysis routine (Bascaran, 1990). Inputs for the simulation include (a) definitions for components, (b) system parameters involved in system synthesis, and (c) the operating environment. System performances, e.g., cycle efficiency, power output, total efficiency and economic benefits, are generated as output. To improve the computational efficiency in function evaluation and predication of performance deviation, Response Surface Methodology (Myers and Montgomery, 1995) is used to create second order response surface models which directly map the design solution into the design performance. Creation of the second order response surface models for this example is facilitated through the use of NORMAN® and is covered thoroughly in our paper (Chen, et al., 1995) and is not repeated here. For all the response surface models, the input design variables X_i are normalized. Therefore, the bounds on all the design variables are represented by $-1= X_i =1$ accordingly.

In Figure 8, the equations for computing the design capability indices for power, efficiency and savings are provided, Eqns. (10), (12), and (14). These equations are based on formulae in Eqns. (4) to (6) for computing the design capability indices when "smaller is better", "nominal is better", and "larger is better". The deviation function Eqn. (15), which represents the difference between the achievable design capability indices (C_{dk-pow}, C_{dk-eff} and C_{dk-sav}) and their targets, is minimized.

3.3 Results of the Compromise DSP Using an Archimedean Deviation Function

For the preceding compromise DSP, the initial study is conducted using an Archimedean deviation function in which all the goals are placed at the same priority level and given equal weights. The goals of maximizing the design capability indices are achieved by minimizing the deviation function comprised of the deviation variables d_i^-. The deviation function, Eqn. (15), thus becomes:

$$Z = 0.33(d_1^-) + 0.33(d_2^-) + 0.33(d_3^-) \qquad (16)$$

where d_i^- measures the deviation between each achievable C_{dk}, i.e., C_{dk-pow}, C_{dk-eff} and C_{dk-sav}, and its target value of 1. The resulting flexible top-level design specifications using the Archimedean deviation function are presented in Table 2. In the table, the deviation of the top-level design specifications correspond to the *range* specified in the problem statement, i.e., ±0.2 after normalizing. A range of values for each top-level design specification is desired to enhance system flexibility for later developments.

Table 2 – Range of Solutions Using the Archimedean Deviation Function

	Flexible Top-Level Design Specifications
X_A (MPA)	2.692 ± 0.207
X_B (K)	454.3 ± 7.00
X_C (K)	537.16 ± 28.00
X_D (kg/s)	2.64E-02 ± 0.35E-02

3.4 Verification of the Results

Our approach is verified in the following ways:

(1) verification of the statistical approach to the evaluation of performance, and

(2) verification of the compromise DSP model by testing the results for different design scenarios.

(3) verification of C_{dk} as a design metric instead of using two separate goals.

<u>(1) Verification of the statistical approach to the evaluation of performance</u>

In this work, we utilize the concept of robust design to capture the variations of design variables (parameters) and the corresponding variations of design performance. The statistical approach (either first-order Taylor expansion or design of experiments) is proposed to evaluate the deviation of performance based on the information of mean and standard deviation. For the example problem, we approximate the deviations of performance (±? y) by (±3σ) based on the assumption of normal distribution and the Central Limit Theorem. We now ask ourselves: (1) *how good is the normality assumption?* and (2) *is three standard deviation (3 s) sufficient enough to capture the total deviation of performance?*

To answer these questions, we conduct a large number of Monto-Carlo simulations (1000 in this case) of design performance across the design space formed by the range of specifications determined by the compromise DSP in Section 3.3 (see Table 2) to study the normality of the performance distributions. Figure 9a is an example of the normality test (illustrated by the Normal Quantille Plot) for performance variable "savings." If the variable follows a normal distribution, then the plot approximates a diagonal straight line. The plot also has the Lillifors confidence bounds which surround the area of normal distribution. It is noted from Figure 9a that the plot of savings is very close to the diagonal straight line. The normality assumption is further tested using the Shapiro-Wilk W test (JMP®, SAS institute). The normality of the performance "power", "efficiency", and "savings" is tested to be valid hypothesis. In Figures 9a and 9b, we also illustrate the differences between the normal curve[5] and the smooth curve[6] for each performance. It is noted that the normal curve captures the spread of the performance distribution caused by the flexible design specifications extremely well.

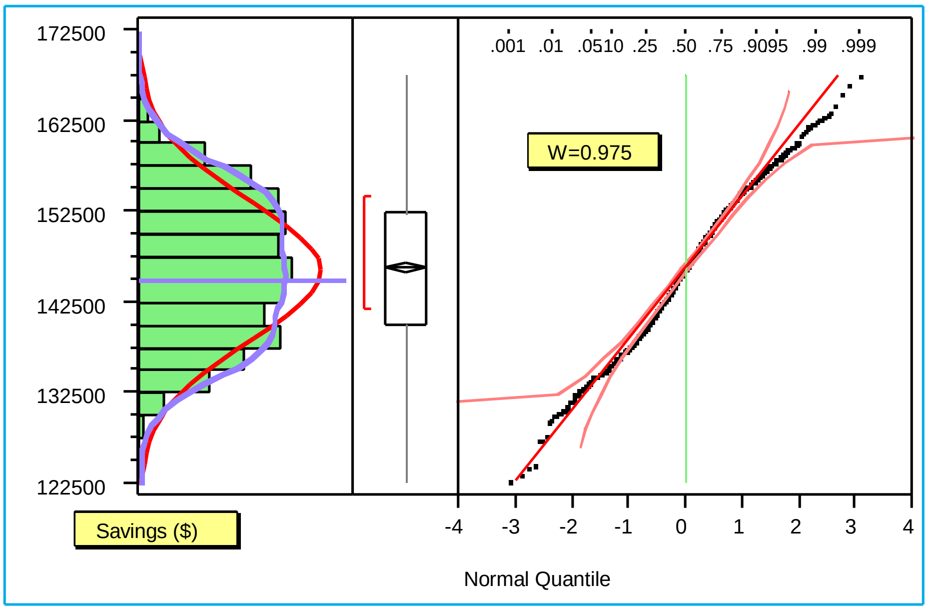

Figure 9a – Normal Quantile Plot of Saving

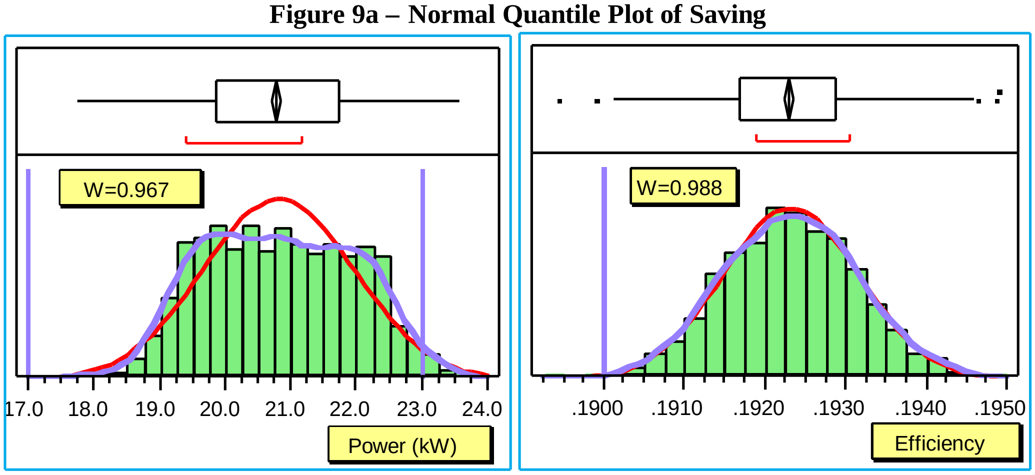

Figure 9b – Plots of Power and Efficiency

Figure 9 – Normality Check of Performance Variables

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[5] Normal Curve superimposes a curve on the histogram based on the mean and standard deviation).

[6] Smooth Curve fits a curve to the histogram using nonparametric density estimation.

In Table 3, a comparison of the deviations of performance obtained from the 1000 Monto-Carlo Simulations and from the estimations using statistical data (μ and σ) is provided. It is noted that the maximum (max) and minimum (min) values of each performance estimated using ($\mu \pm 3\sigma$) are very close to those confirmed from the Monto-Carlo simulations. For "efficiency", the range limits based on ($\mu \pm 3\sigma$) are exactly the same as those confirmed from simulations. This is not surprising because the normality of "efficiency" is the best among the three. For "power" and "savings", the ranges (Min-Max) identified based on ($\mu \pm 3\sigma$) are very close to those obtained from the confirmation simulations. Though slightly bigger, the estimated ranges provide a conservative evaluation of the design capability C_{dk}. Also included in Table 3 are the ranges identified based on ($\mu \pm 6\sigma$) which deviate quite a lot from the reality. We conclude from this study that the technical and economical performance distributions are accurately approximated using normal distributions and that the three standard deviation (3σ) is a good estimation of the total deviation of performance ? y.

Table 3 – Comparisons of Performance Deviations

		Simulation Confirmation	**m ± 3 s**	**m ± 6 s**
Power (kW)	**Max**	23.58	24.28	27.75
	Min	17.77	17.33	13.86
Efficiency	**Max**	0.195	0.195	0.204
	Min	0.189	0.189	0.180
Savings ($)	**Max**	167668	169854	193443
	Min	122800	122677	99088

The design capability indices calculated using the results confirmed from Monto Carlo Simulations are included in Table 4. A comparison of the ranged system performance and the ranged design requirements is provided. As we can see, power varies within the range of 20.675 ± 2.905 kW, i.e., 17.77 to 23.58 kW. The upper bound 23.58 kW will exceed the URL, i.e., 23 kW. In this case, $C_{dk\text{-}pow} = \min\{C_{dl\text{-}pow}, C_{du\text{-}pow}\} = C_{du\text{-}pow} = 0.800$. For efficiency, the ranged design requirement is violated when the performance is at its lower bound, i.e., 0.189. For savings, it is noted that with the current design a large portion of the performance will be outside the ranged design requirements and therefore the design capability index (0.010) is far from its desired value of 1. The capability of a ranged specifications to satisfy a ranged set of design requirements is also graphically illustrated in Figures 9a and 9b in which the specification limits are indicated.

Table 4 – Comparison of System Performance and Ranged Design Requirements

	Performance	**Design Requirement Range**	**Design Capability Index**
Power (kW)	20.675 ± 2.905	[17, 23]	0.800
Efficiency	0.192 ± 0.003	= 0.19	0.667
Savings ($)	145234 ± 22434	= 145,000	0.010

(2) Verification of the compromise DSP model

To verify the validity of incorporating the design capability indices in the compromise DSP model, in addition to our initial study using an Archimedean deviation function, three preemptive deviation functions for Eqn. (15), corresponding to different design scenarios, are used to see whether the changes in the results correspond to the changes in the problem. As shown in Table 5, Scenarios I-III use preemptive deviation functions. In each, the first priority (P_1) is to satisfy a single design requirement, and the second priority (P_2) is assigned to the remaining two design requirements which are both equally weighted. For example, the deviation function in Scenario II is:

$$Z = \{\ P_1[(d_2^-)],\ P_2\ [0.5(d_1^-) + 0.5(d_3^-)]\ \} \tag{17}$$

where d_2^-, the deviation between $C_{dk\text{-}eff}$ and its target of 1, is placed at the first priority level, P_1, and d_1^- and d_3^-, the deviations between $C_{dk\text{-}pow}$ and $C_{dk\text{-}sav}$ and 1 respectively, are equally weighted at the second priority level, P_2. Under Scenario I, $C_{dk\text{-}pow}$ is placed at the first priority, while $C_{dk\text{-}sav}$ is placed at the first priority under Scenario III. These three different design scenarios, in combination with the Archimedean formulation solved in Section 3.3, enable us to determine the extent to which trade-offs occur between the three different goals. In this manner, we gain a better understanding of what the system is capable of achieving. Observations based on the results summarized in Table 5 are provided as follows.

- There are conflicts between satisfying the three design requirements, i.e., power, efficiency and savings, simultaneously. From the results of Scenarios I-III, it is noted that it is possible to almost fully satisfy one of the design requirements when that requirement is placed at the highest priority; $C_{dk\text{-}power}$ = 0.9877 in Scenario I, $C_{dk\text{-}effi}$ =1 in Scenario II and $C_{dk\text{-}savs}$ = 1 in Scenario III. However, since there are conflicts between satisfying the different design requirements simultaneously, the C_{dk}'s achieved for the other two specifications are reduced significantly in all three scenarios. For example, when the design requirement on "power" is at the first priority (Scenario I), the achieved C_{dk} 's for "efficiency" and "savings" are at 0.169 and -0.284, respectively, which are far away from the desired target 1.
- The design solutions of X_A, X_B, X_C, and X_D vary only slightly between Scenarios I and II, while those from Scenario III are quite different. Under Scenario III, the maximum operating pressure X_A is 0.930 MPA, a much lower value compared to the solutions under Scenario I (X_A = 2.695 MPA) and Scenario II (X_A = 2.565 MPA). The fluid flow rate X_D is higher under scenario III (X_D = 0.0318 kg/s) compared to those under the other two preemptive cases.

Table 5 – Compromise DSP Results for Different Design Scenarios

	Scenario I Preemptive	**Scenario II Preemptive**	**Scenario III Preemptive**
Formulation of Deviation Function	$Z = \{ P_1[(d_1^-)]$ $P_2 [0.5(d_2^-)$ $+0.5(d_3^-)] \}$	$Z = \{ P_1[(d_2^-)]$ $P_2 [0.5(d_1^-)$ $+0.5(d_3^-)] \}$	$Z = \{ P_1[(d_3^-)]$ $P_2 [0.5(d_1^-)$ $+0.5(d_2^-)] \}$
System Variables			
X_A (MPA)	2.695	2.565	0.930
X_B (K)	460.305	450.865	450
X_C (K)	561.22	523.46	520
X_D (kg/s)	2.49E-02	2.86E-02	3.18E-02
Mean, μ			
Power (kW)	19.8299	22.5345	23.3044
Efficiency	0.190403	0.192542	0.169873
Savings ($)	139304	158208	163569
Deviation, ? y^2			
Power (kW^2)	8.20843	7.93341	7.05154
Efficiency	5.71E-06	6.36E-06	2.13E-05
Savings ($\$^2$)	4.01E+08	3.88E+08	3.45E+08
C_{dk}			
Power	0.987735	0.165274	-0.114641
Efficiency	0.168794	1.00797	-4.36494
Savings	-0.284262	0.670544	1
Deviation Function Value, Z			
Level I	0.122649E-01	0	0
Level II	1.05773	0.582091	0.557321

The first observation can be interpreted from the mathematical construct of the response surface models for power, efficiency, and savings. It is noted that maximizing these three performance attributes to their desired ranges are conflicting with each other. Comparing the results of the three preemptive formulations (Scenarios I, II and III) with those of the Archimedean formulation (Section 3.3, Table IV), we find the results from the Archimedean formulation (a weighted objective function) represent a tradeoff associated with satisfying the three design requirements simultaneously ($C_{dk\text{-power}} = 0.800$, $C_{dk\text{-effi}} = 0.667$, $C_{dk\text{-savs}} = 0.01$). For the second observation, since satisfying the design requirement of $145,000 for savings is controversial to achieving the other two requirements, therefore when placed at the highest priority level as it is in Scenario III, the results are quite different from the other scenarios. The results match with the physical nature of the problem, in which the decrease of operating pressure X_A and the increase of fluid flow rate X_D are preferred to improve the total savings (see Eqn. (3.3), Table 1, Chen et al. 1996). The compromise DSPs are verified to be effective for making tradeoffs among multiple design requirements.

(3) Verification of C_{dk} as a design metric vs. the two-separate-goals approach

In Section 1.2, we have illustrated graphically the reason why it is not always effective to model the two aspects of robust design, i.e., "bringing the mean on target" and "minimizing the variance", as two separate goals when *satisfying a range of design requirements is the major concern.* We propose to use the design capability index C_{dk} as a design metric to overcome this limitation. To verify the validity of our approach, we compare the results from using these two design metrics for the same problem.

Modeling the two aspects of robust design as two separate goals using the compromise DSP is described in our proposed general robust design procedure (Chen et al. 1996). Using the compromise DSP provided in Table 5, Chen et al. 1996, the target for each design requirement is specified as a particular value instead of a range, e.g., 20 kW for "power", 0.19 for "efficiency", and $145,000 for "savings". Bringing the mean of performance as close as possible to the targets are desired as well as minimizing the deviations of performance. The results for the power irrigation system design using the two-separate-goals approach are illustrated in Table 6 and compared to those obtained from using the design capability index approach. For both compromise DSPs, the Archimedean formulation is used for the deviation function, in which multiple goals are placed at the same priority level with equal weights.

Table 6 – Comparisons of Results using C_{dk} Approach and Two-Separate-Goals Approach

	Cdk Approach	**Two-Separate-Goals Approach**
Design Solution		
X_A (MPA)	2.692 ± 0.207	1.7047 ± 0.207
X_B (K)	454.3 ± 7.00	450.00 ± 7.00
X_C (K)	537.16 ± 28.00	520.00 ± 28.00
X_D (kg/s)	2.64E-02 ± 0.35E-02	2.59E-02 ± 0.35E-02
Performance Deviations (y±? y)		
Power (kW)	20.675 ± 2.905	19.952 ± 2.719
Efficiency	0.192 ± 0.003	0.183 ± 0.0035
Savings ($)	145234 ± 22434	140143 ± 19000
Normalized Deviations		
Dev. from target mean	0.0684	0.0727
Dev. from target var.	0.2491	0.1667
Total Deviations	**0.3175**	**0.2394**
C_{dk}		
Power	0.80	1.09
Efficiency	0.67	-2.00
Savings	0.01	-0.26
Total Deviations from Cdk=1	**1.52**	**4.26**

It is noted from Table 6 that the mean locations of the design solutions are different under the two formulations, with the difference between the values of operating pressure X_A the biggest, i.e., 2.692 MPA vs. 1.7047 MPA. Corresponding to the same range of deviations of design solutions, the performance deviations (y±? y) are compared. It is observed that, for each performance attribute, the results obtained from the two design formulations are different in both the mean value (y) and the deviations (? y). Though the performance based on the C_{dk} approach result a smaller total normalized deviations (i.e., 0.0684) from the target means than those based on the two-separate-goals approach (i.e., 0.0727), the total normalized deviations from the target variances is much bigger (0.2491 vs. 0.1667). After summing these two types of deviations, the results from using the two-separate-goals approach is more favorable (0.2394 vs. 0.3175), which is consistent with the goal formulation. Similarly, the achieved design capability indices are compared between these two groups of designs. It is observed that the results from using the C_{dk} approach is more favorable this time (1.52 vs. 4.26 total deviations from goal targets). The aforementioned observations indicate that the solutions are consistent with our intentions when setting the design metrics.

We also find that the deviations of performance are usually reduced to a smaller value when the two-separate-goals approach for robust design is applied. This is consistent with the robust design principle, under which a good design is considered to the one with the smallest deviation in addition to the "mean on target". When satisfying a range of solutions is the major concern, the percentage overlap between the *range* of performance and the *range* of requirement becomes more critical rather than the deviations of performance itself.

4. CLOSURE

A systematic approach for efficiently determining *ranged* top-level design specifications, corresponding to a *family* of design concepts, to satisfy a *range* of design requirements is presented. Our method is rooted in the concept of robust design, one of the existing approaches used to the study of variation in product and process development. The concept of process capability indices is extended to develop design capability indices which are used to assess the capability of the *ranged* top-level design specifications to satisfy *ranged* design requirements. Our approach appears to have the following three major advantages:

- *Efficient for evaluating a family of designs:* It is always a tedious task to check whether a family of designs can satisfy the design requirements. The evaluation process becomes explosive when a large number of design variables must be considered, particularly when these design variables are ranges and not discrete solutions. In a complex domain, the principles of fuzzy set theory and the interval method are difficult to implement because of their limitations of working with inexplicit or closed-form systems analyses. By modeling the problem using principles from robust design, the process is expedited by predicting the *spread* of the design performance by approximating the performance variance. As shown in Figure 9 and Table 3, the variation of system performance corresponding to the flexible top-level design specifications are computed using the information of mean and standard deviation. Our study shows that the performance in our study follows the normality very closely. This is consistent with the Central Limit Theorem. Meanwhile, the three standard deviation (3σ) is a good estimation of the total deviation of performance ? y. In Table 2, the statistical interpretation of different k values used for calculating the performance deviation is provided.

- *Easy to compute and easy to understand:* As a unitless value, the design capability index, C_{dk}, is easy to compute and is more intuitive than some existing measures for assessing the capability of the top-level design specifications to satisfy a ranged set of design requirements. For example, a negative value for C_{dk} indicates that the mean of the system performance is outside of the range of specifications. As shown in Tables 2 and 3, the closer C_{dk} is to 1, the greater the portion of the design requirement that is satisfied. As a unitless measure, it is easy to use C_{dk} for comparing the goodness of different sets of top-level design specifications.
- *Incorporate multiple aspects in quality improvement:* Different from other approaches to satisfying a ranged set of design requirements, our approach incorporates quality considerations by considering the location of the mean of performance distribution as well as the variation. Design capability indices are

developed for various situations such as "nominal is best", "smaller is better", and "larger is better".

Although our approach is based on concepts from statistical process control and robust design, it is not necessary that the deviations, σ_i, of the top-level design specifications and/or the design performance be statistical. This approach can easily be adapted to problems with imprecise information which is quantifiable with fuzzy numbers for example. The essence of using a design capability index is to predict the *spread* of the design performance so that the capability of a *family of designs* to satisfy a *range of design requirements* can be assessed. In this manner, designs are more flexible in the later stages of design and can be readily adapted to changing design requirements allowing companies to respond more quickly and at lower costs than their competitors.

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Given

An alternative to be improved. Assumptions used to model the domain of interest.

The system parameters:

n	number of system variables	q	inequality constraints
p + q	number of system constraints	m	number of system goals
$g_i(\mathbf{X})$	system constraint function:		
$f_k(d_i)$	function of deviation variables to be minimized at priority level k for the preemptive case.		

Find

$X_i \qquad i = 1, \ldots, n$

$d_i^-, d_i^+ \qquad i = 1, \ldots, m$

Satisfy

System constraints (linear, nonlinear)

$g_i(\mathbf{X}) = 0 ; \qquad i = 1, \ldots, p$

$g_i(\mathbf{X}) = 0 ; \qquad i = p+1, \ldots, p+q$

System goals (linear, nonlinear)

$A_i(\mathbf{X}) + d_i^- - d_i^+ = G_i ; i = 1, \ldots, m$

Bounds

$X_i^{min} = X_i = X_i^{max} ; \qquad i = 1, \ldots, n$

$d_i^- , d_i^+ = 0 ; \qquad i = 1, \ldots, m$

$d_i^- \cdot d_i^+ = 0 ; \qquad i = 1, \ldots, m$

Minimize

Preemptive deviation function (lexicographic minimum)

$\mathbf{Z} = [f_1(d_i^- , d_i^+), \ldots, f_k(d_i^- , d_i^+)]$

Figure 1 - Mathematical Form of a Compromise DSP

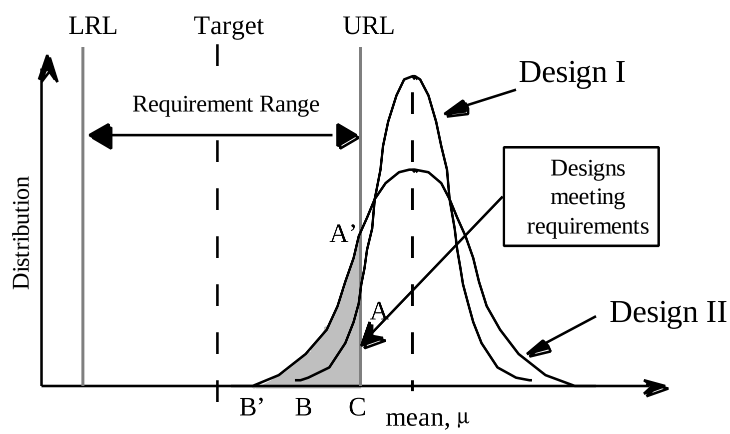


Figure 2 - Quality Performance Distribution of Two Designs

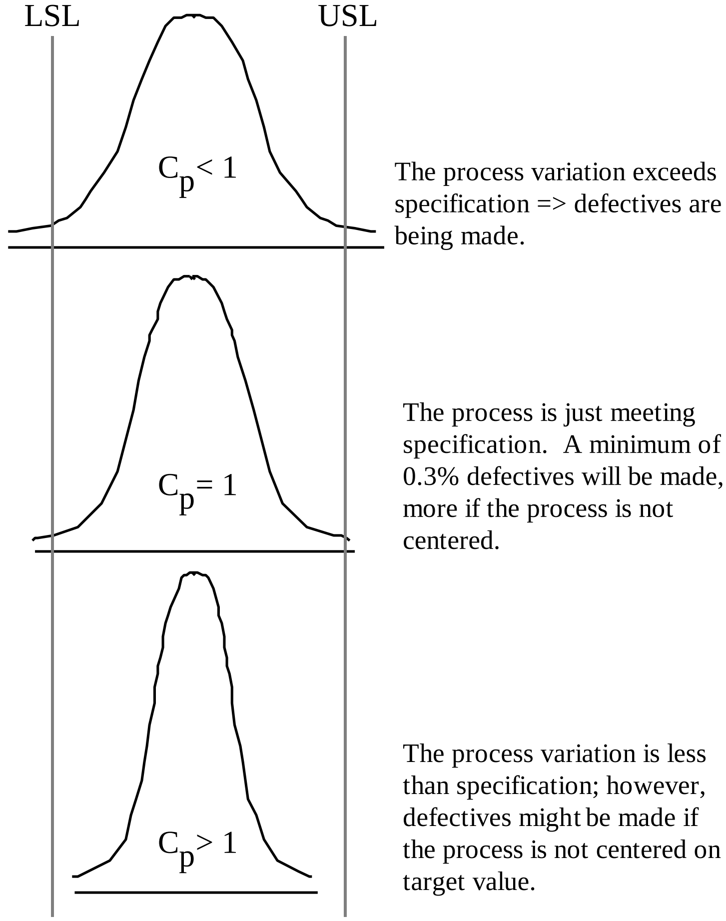


Figure 3 – Relationship Between Process Capability Index and Customer Specifications (Brassard and Ritter, 1994)

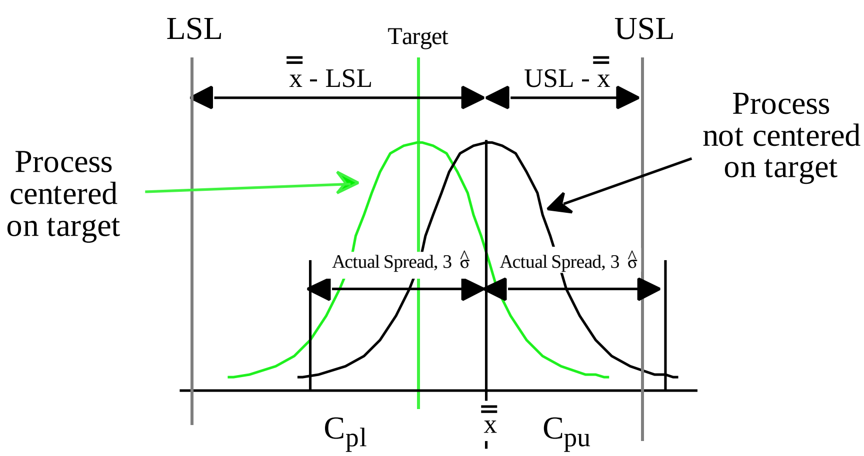
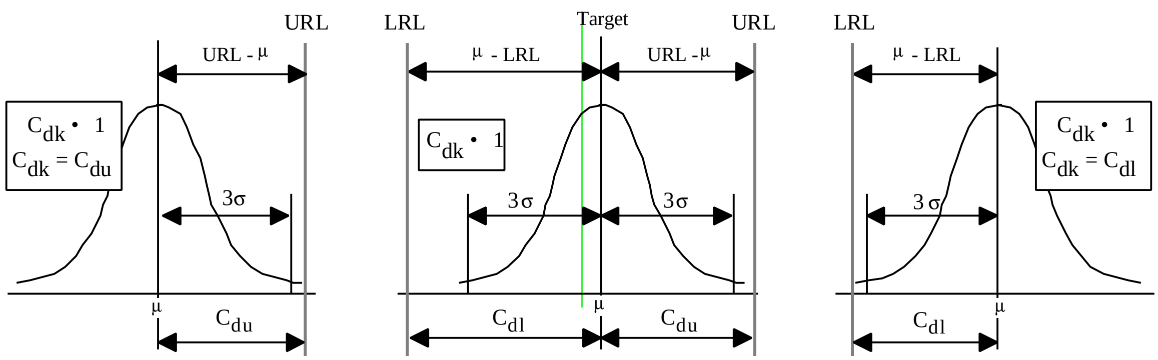


Figure 4 – Process Capability Indices for Process Mean not Centered on Target
(Brassard and Ritter 1994)



<u>Smaller is Better</u>

- Target below URL
- All variants below URL
- Desire C_{dk} • 1, $C_{dk} = C_{du}$

<u>Nominal is Better</u>

- Target between URL and LRL
- All variants between URL and LRL
- Desire C_{dk} • 1, $C_{dk} = \min\{C_{dl}, C_{du}\}$

<u>Larger is Better</u>

- Target above LRL
- All variants above LRL
- Desire C_{dk} • 1, $C_{dk} = C_{dl}$

Figure 5 – Design Capability Indices

Given

Functions **y** including those ranged design requirements which are constraints, $g_i(\mathbf{X})$, and those which are objectives, $A_j(\mathbf{X})$

Deviations of the control variables, $\sigma_{\mathbf{X}}$ or ? x

Target *ranges* for the design requirements, URL_i and LRL_i

Find

The location of the mean of the control variables $\mu_{\mathbf{X}}$

Satisfy

Constraints: $C_{dk\text{-constraints}} = 1.$ (7)

Goals: $C_{dk\text{-objectives}} + d_i^- - d_i^+ = 1$ (8)

Bounds:

$d_i^-, d_i^+ = 0;$ $i = 1, \ldots, m$

$d_i^- \cdot d_i^+ = 0;$ $i = 1, \ldots, m$

Minimize Deviation Function $Z=[f_1(d_i^-, ..., d_i^+)...f_k(d_i^-, ..., d_i^+)]$

Figure 6 – The Compromise DSP Particularized for Designing Flexible Top-Level Design Specifications Using Design Capability Indices

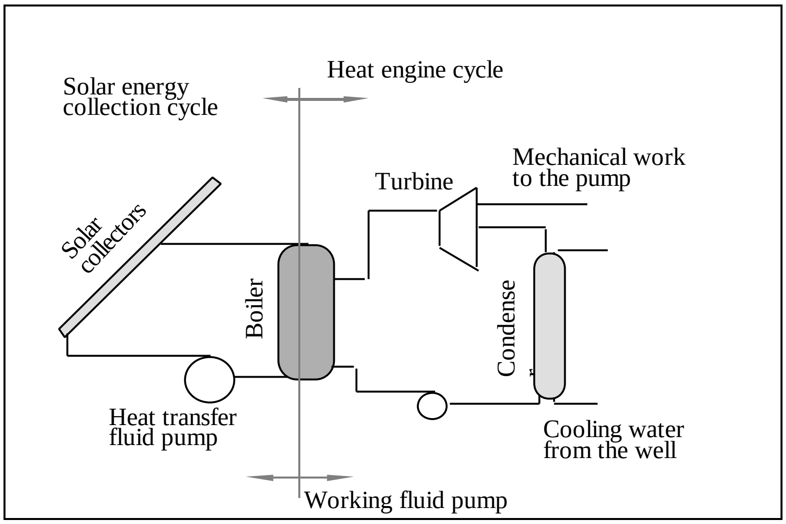


Figure 7 - Solar Powered Irrigation System

Given

- Second order response surface models for Power, Overall Efficiency, and Savings as functions of X_A, X_B , X_C, X_D, i.e., POW(**X**), EFF(**X**), and SAV(**X**).
- ? X_A = ? X_B = ? X_C = ? X_D = 0.2 (normalized)
- The target ranges for the design requirements:

  Power: 17 kW = T_{pow} = 23 kW (nominal is better)

  Efficiency T_{eff} = 19% (larger is better)

  Savings T_{sav} = $145,000 (larger is better)
- The targets for the design capability indices: C_{dk}-pow = C_{dk}-eff = C_{dk}-sav = 1.

Find

- The mean of the top-level design specifications

X_A,	Normalized cycle maximum pressure	n.m.u.
X_B,	Normalized cycle maximum temperature	n.m.u.
X_C,	Normalized collector maximum temperature	n.m.u.
X_D,	Normalized working fluid flow rate	n.m.u.

- The values of deviation variables associated with the goals d_i^-, d_i^+ n.m.u. (i =1, 3)

Satisfy

- The goals:

  Achieve power within the range of specification (nominal is better)

  $C_{dk-pow} + d_1^- - d_1^+ = 1,$ (9)

  $C_{dk-pow} = \min \{C_{dl-pow}, C_{du-pow}\}$ where

  $C_{dl-pow} = \{POW(\mathbf{X}) - 17\}/?\,POW(\mathbf{X})$, & (10a)

  $C_{du-pow} = \{23 - POW(\mathbf{X})\}/?\,POW(\mathbf{X})$ (10b)

  Achieve the efficiency within the range of specification (the larger is better)

  $C_{dk-eff} + d_2^- - d_2^+ = 1,$ (11)

  $C_{dk-eff} = C_{dl-eff} = \{EFF(\mathbf{X}) - 0.19\}/?\,EFF(\mathbf{X})$ (12)

  Achieve the savings within the range of specification (the larger is better)

  $C_{dk-sav} + d_3^- - d_3^+ = 1,$ (13)

  $C_{dk-sav} = C_{dl-sav} = \{SAV(\mathbf{X}) - 145{,}000\}/?\,SAV(\mathbf{X})$ (14)
- Bounds on the design variables

  $-1 = X_A = 1$

  $-1 = X_B = 1$

  $-1 = X_C = 1$

  $-1 = X_D = 1$
- $d_i^+ \cdot d_i^- = 0$, with $d_i^+, d_i^- = 0$

Minimize

- The sum of the deviation variables associated with:
  - the design capability index for power, C_{dk-pow}, d_1^-
  - the design capability index for efficiency, C_{dk-eff}, d_2^-
  - the design capability index for savings, C_{dk-sav}, d_3^-

  $Z = [\ f_1(d_1^-),\ f_2(d_2^-),\ f_3(d_3^-)]$ (15)

Figure 8 – The Compromise DSP for Designing Flexible Top-Level Design Specifications Using Design Capability Indices

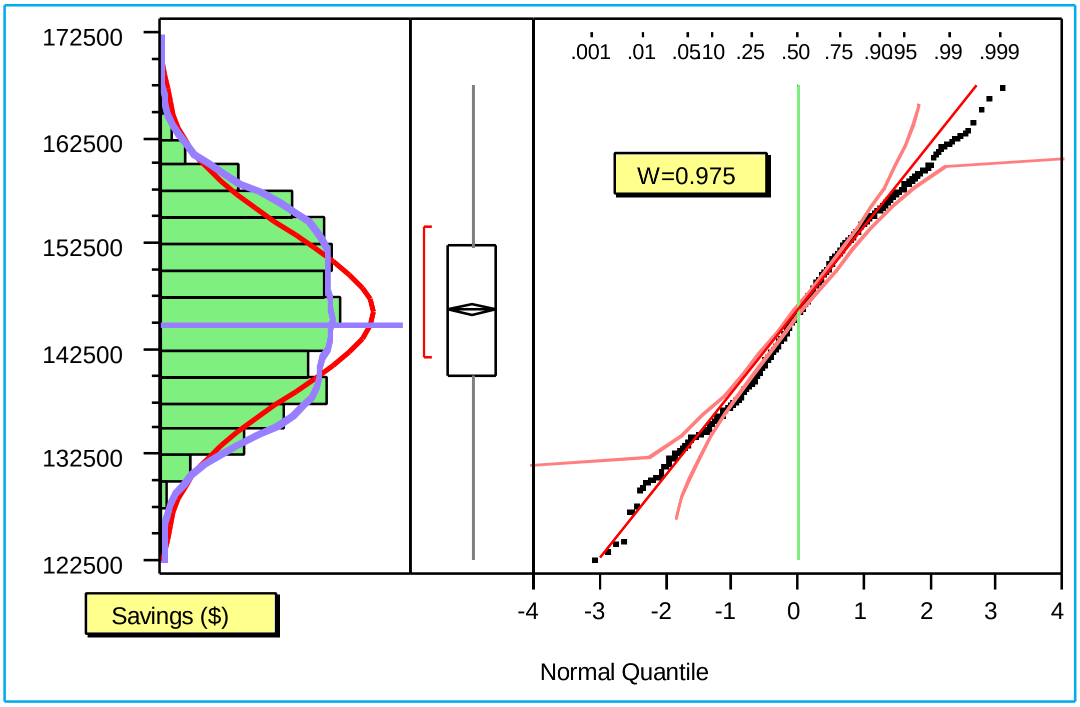


Figure 9a – Normal Quantile Plot of Saving

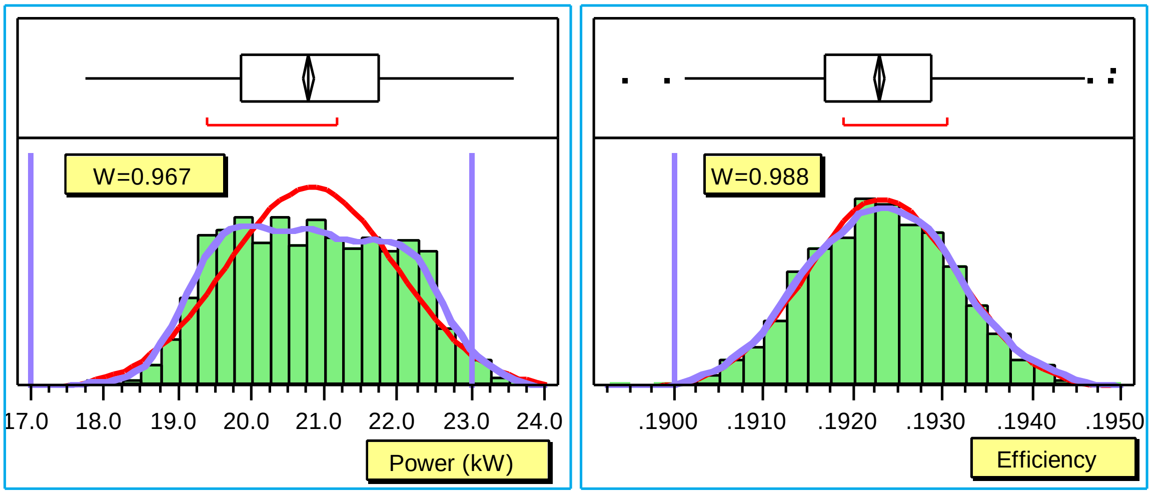


Figure 9b – Plots of Power and Efficiency

Figure 9 – Normality Check of Performance Variables

Table 1 – The Relation of k to the Percentage of Performance Conforming to Requirements

k (number of standard deviations)	**1**	**2**	**3**	**4**
% of performance conforming to requirements	84.13	97.725	99.865	99.9968

Note: k is different from C_{dk}

Table 2 – Range of Solutions Using the Archimedean Deviation Function

	Flexible Top-Level Design Specifications
X_A (MPA)	2.692 ± 0.207
X_B (K)	454.3 ± 7.00
X_C (K)	537.16 ± 28.00
X_D (kg/s)	2.64E-02 ± 0.35E-02

Table 3 – Comparisons of Performance Deviations

		Simulation Confirmation	**m ± 3s**	**m ± 6s**
Power (kW)	**Max**	23.58	24.28	27.75
	Min	17.77	17.33	13.86
Efficiency	**Max**	0.195	0.195	0.204
	Min	0.189	0.189	0.180
Savings ($)	**Max**	167668	169854	193443
	Min	122800	122677	99088

Table 4 – Comparison of System Performance and Ranged Design Requirements

	Performance	Design Requirement Range	Design Capability Index
Power (kW)	20.675 ± 2.905	[17, 23]	0.800
Efficiency	0.192 ± 0.003	= 0.19	0.667
Savings ($)	145234 ± 22434	= 145,000	0.010

Table 5 – Compromise DSP Results for Different Design Scenarios

	Scenario I Preemptive	Scenario II Preemptive	Scenario III Preemptive
Formulation of Deviation Function	$Z = \{ P_1[(d_1^-)]$ $P_2\ [0.5(d_2^-)$ $+0.5(d_3^-)]\ \}$	$Z = \{ P_1[(d_2^-)]$ $P_2\ [0.5(d_1^-)$ $+0.5(d_3^-)]\ \}$	$Z = \{ P_1[(d_3^-)]$ $P_2\ [0.5(d_1^-)$ $+0.5(d_2^-)]\ \}$
System Variables			
X_A (MPA)	2.695	2.565	0.930
X_B (K)	460.305	450.865	450
X_C (K)	561.22	523.46	520
X_D (kg/s)	2.49E-02	2.86E-02	3.18E-02
Mean, μ			
Power (kW)	19.8299	22.5345	23.3044
Efficiency	0.190403	0.192542	0.169873
Savings ($)	139304	158208	163569
Deviation, ? y^2			
Power (kW^2)	8.20843	7.93341	7.05154
Efficiency	5.71E-06	6.36E-06	2.13E-05
Savings ($\$^2$)	4.01E+08	3.88E+08	3.45E+08
$\mathbf{C_{dk}}$			
Power	0.987735	0.165274	-0.114641
Efficiency	0.168794	1.00797	-4.36494
Savings	-0.284262	0.670544	1
Deviation Function Value, Z			
Level I	0.122649E-01	0	0
Level II	1.05773	0.582091	0.557321

Table 6 – Comparisons of Results using C_{dk} Approach and Two-Separate-Goals Approach

	Cdk Approach	**Two-Separate-Goals Approach**
Design Solution		
X_A (MPA)	2.692 ± 0.207	1.7047 ± 0.207
X_B (K)	454.3 ± 7.00	450.00 ± 7.00
X_C (K)	537.16 ± 28.00	520.00 ± 28.00
X_D (kg/s)	2.64E-02 ± 0.35E-02	2.59E-02 ± 0.35E-02
Performance Deviations (y±? y)		
Power (kW)	20.675 ± 2.905	19.952 ± 2.719
Efficiency	0.192 ± 0.003	0.183 ± 0.0035
Savings ($)	145234 ± 22434	140143 ± 19000
Normalized Deviations		
Dev. from target mean	0.0684	0.0727
Dev. from target var.	0.2491	0.1667
Total Deviations	**0.3175**	**0.2394**
C_{dk}		
Power	0.80	1.09
Efficiency	0.67	-2.00
Savings	0.01	-0.26
Total Deviations from Cdk=1	**1.52**	**4.26**