MODEL VALIDATION VIA UNCERTAINTY PROPAGATION AND DATA TRANSFORMATIONS

Wei Chen*

Integrated Design Automation Laboratory (IDEAL)

Department of Mechanical Engineering

Northwestern University

Lusine Baghdasaryan

Department of Mechanical & Industrial Engineering

University of Illinois at Chicago

Thaweepat Buranathiti and Jian Cao

Advanced Materials Processing Laboratory

Department of Mechanical Engineering

Northwestern University

Evanston, Illinois

* Corresponding Author, Mechanical Engineering, Northwestern University
2145 Sheridan Road, Tech B224, Evanston, IL 60208-3111; Phone: (847) 491-7019, Fax: (847) 491-3915; email: weichen@northwestern.edu.

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Abstract

Model validation has become a primary means to evaluate accuracy and reliability of computational simulations in engineering design. Due to uncertainties involved in modeling, manufacturing processes, and measurement systems, the assessment of the validity of a modeling approach must be conducted based on stochastic measurements to provide designers with the confidence of using a model. In this paper, a generic model validation methodology via uncertainty propagation and data transformations is presented. The approach reduces the number of physical tests at each design setting to one by shifting the evaluation effort to uncertainty propagation of the computational model. Response surface methodology is used to create metamodels as less costly approximations of simulation models for the uncertainty propagation. Methods for validating models with both normal and nonnormal response distributions are proposed. The methodology is illustrated with the examination of the validity of two finite element analysis models for predicting springback angles in a sample flanging process.

Key words: Model validation, Uncertainty propagation, Response surface models, Sheet metal forming, Data transformations.

1. INTRODUCTION

The increased dependence on using computer simulation models in engineering design arises a critical issue of confidence in modeling and simulation accuracy. Model verification and validation are the primary methods for building and quantifying confidence, as well as for the demonstration of correctness of a model [1], [2]. Briefly, *model verification* is the assessment of the solution accuracy of a mathematical model. Model validation, on the other hand, is the assessment of how accurately the mathematical model represents the real world application [3]. Thus, in verification, the relationship of the simulation to the real world is not an issue, while in validation, the relationship between the virtual (computation) and the real world, i.e., experimental data, is the issue.

One limitation of the existing model validation approaches is that they are restricted to the validation at a particular design setting. There is no guarantee that the conclusion can be extended over the entire design space. In addition, model validations are frequently based on comparisons between the output from *deterministic simulations* and that from single or repeated experiments. The existing statistical approaches, for which the physical experiment has to be repeated a sufficient number of independent times, is not practical for many applications, simply due to the cost and time commitment associated with experiments. Furthermore, deterministic simulations for model validation do not consider uncertainty at all. Although recent model validation approaches propose to shift the effort to propagating the uncertainty in model predictions, which implies that a model validation should include all relevant sources of uncertainties, little work has been accomplished in this area [1], [2], [4]. Since realistic mathematical models should contemplate uncertainties, the assessment of the validity of a modeling approach must be conducted based on stochastic measurements to provide designers with the confidence of using a model.

Traditionally, a model has been considered valid if it reproduces the results with adequate accuracy. The two traditional model validation approaches are: 1) subjective and 2) quantitative comparisons of model predictions and experimental observations. Subjective comparisons are through visual inspection of x-y plots, scatter plots and contour plots. Though they show the trend in data over time and space, subjective comparisons depend on graphical details. Quantitative comparisons, including the measures of correlation coefficient and other weighted and non-weighted norms, quantify the “distance” but become very subjective when defining what magnitudes of the measures are acceptable. To quantify model validity from a stochastic perspective, researchers have proposed various statistical inference techniques, such as χ^2 test on residuals between model and experimental results [5]. These statistical inferences require multiple evaluations of the model and experiments, and many assumptions that are difficult to satisfy. Therefore, there is a need for a model validation approach that takes the least amount of statistical assumptions and requires the minimum number of physical experiments.

In this paper, we present a rigorous and practical approach for model validation (*Model Validation via Uncertainty Propagation*) that utilizes the knowledge of system variations along with computationally efficient uncertainty propagation techniques to provide a stochastic assessment of the validity of a modeling approach for a specified design space. Various sources of uncertainties in modeling and in physical tests are evaluated and the number of physical testing at each design setting is reduced to ONE. Response surface methodology is used to create a metamodel of an original simulation model, and therefore, the computational effort for uncertainty propagation is reduced. By employing data transformations, the approach can also be applied to the response distributions that are non-normal. This helps us represent the data in a form that satisfies the assumptions underlying the r^2 method, an approach used in this work to determine whether the results from physical experiments fall inside or outside of the prespecified confidence region. Even though the proposed methodology is demonstrated for validating two finite-element models for simulating sheet metal forming, namely a flanging process, it can be generalized to other engineering problems.

This paper is organized as follows. In Section 2, the technical background of this research is provided. The major types of uncertainties in modeling are first introduced and classified into three categories. Existing techniques on uncertainty propagation are then reviewed, the background of the response surface methodology and statistical data transformations are provided. Our proposed model validation approach is described in Section 3. In Section 4, our proposed approach is demonstrated using a case study in sheet metal forming by examining two finite element based models. Finally, conclusions are provided in Section 5.

2. TECHNICAL BACKGROUND

2.1 CLASSIFICATION OF UNCERTAINTIES

Various types of uncertainties exist in any physical system and in its modeling process and can affect the final experimental or predicted system response. Different ways of classifying uncertainties have been seen in the literature [6]-[9]. In this work, we classify uncertainties into three major categories:

- Type I: Uncertainty associated with the inherent *variation* in the physical system or environment that is under consideration. For example, uncertainty associated with incoming material, initial part geometry, tooling setup, process setup, and operating environment.
- Type II: Uncertainty associated with *deficiency* in any phase or activity of the simulation process that originates in lack of system knowledge. For example, uncertainty associated

with the lack of knowledge in the laws describing the behavior of the system under various conditions, etc.

- Type III: Uncertainty associated with *error* that belongs to recognizable deficiency but is not due to lack of knowledge. For example, uncertainty associated with the limitations of numerical methods used to construct simulation models.

When providing the stochastic assessment of model validity, all these three types of uncertainties should be taken into account.

2.2 TECHNIQUES FOR UNCERTAINTY PROPAGATION

The use of an analysis approach to estimate the effect of uncertainties on model prediction is referred to as uncertainty propagation. Several categories of methods exist in the literature. The first category is the conventional sample-based approach such as Monte Carlo Simulations (MCS). Although alternative sampling techniques such as Quasi Monte Carlo Simulations including Halton sequence [10], Hammersley sequence [11], and Latin Supercube Sampling [12] have been proposed, none of these techniques are computationally feasible for problems that require complex computer simulations, each taking at least a few minutes or even hours or days. Validating a modeling approach at multiple design settings becomes computationally infeasible. The second category of uncertainty propagation approach is based on sensitivity analysis. Most of these methods only provide the information of mean and variance based on approximations. The level of accuracy is not sufficient for applications in model validation. Here, we propose to use a response surface model (or metamodel) to replace the numerical model for uncertainty propagation. The response surface model is generated as a function of design variables and parameters. Monte Carlo Simulations are later performed using the response surface model as a surrogate of the original numerical program. Details of response surface methodologies are provided in the next section.

2.3 RESPONSE SURFACE METHODOLOGIES

Response surface methodologies are well known approaches for constructing approximation models based on either physical experiments or computer experiments (simulations) [13]. Our interest in this work is the latter where computer experiments are conducted by simulating the to-be-validated model to build response surface models. They are often referred to as metamodels as they provide a "model of the model" [14], replacing the expensive simulation models during the design and optimization process. In this paper, response surface models based on simulation results from finite element models are constructed and tested for model validation by using two response surface modeling methods: Polynomial Regression (PR) and Kriging Methods (KG).

Polynomial regression models have been applied by a number of researchers [15], [16], in designing complex engineering systems. In spite of the advantage obtained from the smoothing capability of polynomial regression for noisy functions, there is always a drawback when applying PR to model highly nonlinear behaviors. Higher-order polynomials can be used; however, instabilities may arise [17], or it may be too difficult to take sufficient sample data to estimate all coefficients in the polynomial equation, particularly in large dimensions.

A Kriging model [18] postulates a combination of a polynomial model and departures from it, where the latter is assumed to be a realization of a stochastic process with a zero mean and a spatial correlation function. A variety of correlation functions can be chosen [19], the Gaussian correlation function proposed in is the most frequently used. The Kriging method is extremely flexible to capture any type of nonlinear behaviors due to the wide range of the correlation functions. The major disadvantages of the Kriging process are that model construction can be very time-consuming and could be ill-conditioned [20].

In our earlier works, the advantages and limitations of various metamodeling techniques have been examined using multiple modeling criteria and multiple test problems [21], [22]. Our strategy in this work is to first fit a second-order polynomial model. If the accuracy is not satisfactory, the Kriging method will be employed; otherwise the low-cost polynomial model will be used for uncertainty propagation and model validation.

2.4. DATA TRANSFORMATIONS

Many statistical tests are based on the assumption of normality. When data deviate from normality, an appropriate transformation can often yield a data set that does follow approximately a normal distribution [23]. Generally, response distributions obtained from uncertainty propagation at multiple design points may not be normal. Data transformations are therefore employed in order to use the proposed validation approach that is based on the normality assumption.

One of the most common and simplistic parametric transformation families studied by Tukey [24] and later modified by Box and Cox [25] is:

$$Z^{(\lambda)} \quad = \frac{Z^{\lambda} - 1}{\lambda}, \qquad \lambda \neq 0; \qquad\qquad (1)$$
$$\qquad = \log Z, \qquad \lambda = 0$$

where λ is the transformation parameter. For different values of λ different transformations are obtained. When λ = 1, no transformation occurs. When λ <1, the transformation makes the variance of residuals smaller at large Z's, and makes it larger at small Z's. When λ>1, it has the opposite effects of λ <1. When λ = 0, natural logarithm is used (see equation 1).

The Box-Cox transformation in equation (1), called the power transformation, is only appropriate for positive data. Hinkley [26], Manly [27], John and Draper [28], and Yeo and Johnson [29] proposed alternative families of transformations that can be used to compensate the restrictions on Z, to obtain an approximate symmetry or to make the distribution closer to normal.

The parameters of a transformation, e.g. λ, can be selected through a trial and error approach until good normal probability plots are obtained, through optimization based on Maximum Likelihood estimation or Bayesian estimation [25], likelihood ratio test [30] , or the use of M-estimators [31], etc. Atkinson and Riani [32] and Krzanowski [33] discussed some aspects of multivariate data transformations in more detail.

It is often more useful to apply transformations of predictor (model input) variables, along with the transformations of the dependent (model output) variables. Box and Tidwell [35] provided an iterative procedure to estimate appropriate transformations of the original model inputs.

Atkinson and Riani [32] discussed different models and reasons for what transformations of predictor variables can be applied. It should also be noted that applying the existing transformation techniques may have little effect if the values of the response are far from zero and the scatter in the observations is relatively small (in other words, the ratio of the largest to smallest observation should not be too close to one) [30].

3. OUR PROPOSED MODEL VALIDATION APPROACH

3.1 General Description of the Approach

Our proposed model validation approach is illustrated in Figure 1. The whole process includes four major phases, in which Phase II and Phase III can be implemented in parallel. Phase I is the *Problem Setup* stage. Here, uncertainties of all types described in Section 2.1 are investigated; probabilistic descriptions of model inputs are established. With the aim of model validation over a design space rather than at a single design point, sample design settings, represented by x^i, $i = 1 \ldots n$, are formed using different combinations of values of design variables. The sampling can be based on the knowledge of critical combinations of design variables at different levels, the standard statistical techniques such as Design of Experiments (DOE) [13], or other methods for efficient data sampling (e.g., optimal Latin Hyper Cube [37]). These techniques will be useful in reducing the size of samples when the number of design variables considered is large.

Phase II is the (Physical) *Experimental* stage. *One of the cornerstones of this proposed approach is the minimum number of physical tests required.* Physical experiments will be performed only **once** at each design setting identified in Phase I. Measurements are taken for the model responses that are of interest. The results of experiments are denoted as Y^i, $i = 1,\ldots, n$. Errors of measurements are predicted.

Phase III (*Model Uncertainty Propagation*) is the stage for uncertainty propagation based on the to-be-tested (computational) model. For computationally expensive models, we propose to first construct a response surface model based on samples of numerical simulation results. Techniques introduced in Section 2.3 can be applied here. Next, the total uncertainty of the response prediction is analyzed using the response surface model through Monte Carlo Simulations (MCS) introduced as following. It should be noted that for model validation at multiple design points, uncertainty of the response prediction needs to be evaluated for each of the design points as identified in Phase I.

The uncertainty of the (computational) model prediction can be evaluated by uncertainty propagation using MCS applied to the metamodel (in this case, response surface model), following the uncertainty descriptions identified in Phase I. When a sufficient number of simulations are performed, the MCS is robust in a sense that it provides good estimates of uncertainty in the predicted parameters, no matter whether the model is highly nonlinear or not. The MCS also provides estimates of the shape of the probability density functions (pdf), which are used further in Phase IV for model validation. If the normality checks for the pdf's from MCS are rejected, we propose to apply data transformations to data from the simulation models before constructing response surface models so that the transformed distributions become normal.

Insert Figure 1. Procedure for Model Validation

Phase IV is the *Model Validation* phase, when the stochastic assessment of model validity is drawn based on the comparisons of the physical experimental results from Phase II and the computational results from Phase III. The strategies introduced by Hills et al. [1] are followed here. As Hills' validation criterion for multiple design settings is applicable only for normal response distributions, data transformation is proposed in this work to extend the applicability of the proposed approach to non-normal response distributions. Details of model validation strategies for single and multiple design points, procedures for data transformation, and accounting various sources of errors are discussed next.

3.2 Model validation at a single design point

Hills' method states that for a given confidence bound (say 100*(1-α%)), if the physical experiment falls within the performance range obtained from the computer model (here, the probability density function (pdf) obtained from the MCS in Phase III), it indicates that the model is consistent with the experimental result (however, we can not say the model is valid for the confidence bound). On the other hand, if one physical experiment is outside of the performance range, then we would reject the model for that specified confidence bound (100*(1-α%). *Our strategy of model validation is to identify at which critical limit of confidence level (p-value) the physical experiment falls exactly at the boundary of the performance range obtained from the computer model* (see Figure 2). Therefore, if the given confidence level is lower than the critical limit of confidence level, the model will be rejected, and vice versa.

***Inert Figure 2. Model Validation for a Single Design Point*.**

As shown illustratively in Figure 2, the probability density function describes the distribution of a response based on the (computational) model for the given uncertainty description at a single design point. The confidence limit with which one cannot reject the simulation model is the area under the pdf curve that bounds exactly on the physical experiment, includes the mean of the pdf, and excludes the two equally sized tails that depend on the location of the physical experiment. If the confidence limit is identified as γ% which is smaller than the given confidence bound (e.g., $100*(1-\alpha)$% in Figure 2), we cannot reject the model for an experiment that falls on the boundary of γ%; otherwise we can reject the model since the physical experiment falls outside the distribution range. When a model is rejected, it indicates that a new model needs to be constructed and the whole procedure of model validation should be carried out again. It should be noted that since stochastic assessments are provided for model validity, there are certain risks associated with the error of hypothesis testing [38]. In our case, the false positive error (commonly referred to as a Type I error) is the error of rejecting a model while the true state is that the model is indeed valid. The probability of leading to this outcome is α%. We note that providing a higher confidence bound (lower α%) would widen our acceptance region, while it will reduce our chances of rejecting a valid model, it would also increase our chance of accepting an invalid model, i.e., increasing the probability of making the false negative error (referred to as a Type II error). Indications of Type I and Type II errors in model validation were discussed by Oberkampf and Trucano [2], where they related the Type I error to a model builder's risk and Type II error to model users' risk.

3.3 Model validation at multiple design settings

When a model needs to be validated at multiple design settings, the experimental results need to be compared against the joint probability distributions of a response at multiple design settings. The probability distributions of y^i at multiple design settings (n) are used to generate the joint probability distributions (multidimensional histogram). The contours of the joint probability distributions are used to define the boundary of a given confidence level for model validation and compared with the results from physical tests. Provided in Figure 3 is an illustrative example of model validation for a problem with two physical tests (corresponding to two design settings). The joint pdf of y^1 and y^2 is first obtained for the same response, and then the boundary with 1-α confidence level is determined by the iso-count contour that contains 100(1-α)% samples of Monte Carlo Simulations conducted over the RSM. Theoretically, if the experimental result Y in an n-dimensional space (n = 2 in this example) falls within the boundary, it indicates that we cannot reject the model with a confidence level of (1-α). If the point falls outside of the boundary, then we can reject the model with a confidence level of (1-α). Note the results of single experiments at multiple design settings now become a single point in the multivariate histogram space (see Figure 3).

Insert Figure 3. Model validation at two design settings

For multivariate distributions symmetric about their means, contours of constant probability are given by ellipses determined with r^2. r^2, which can be thought of as a square of the weighted distance of the physical experiments from the multivariate mean, can be related to normal probability through the chi-square distribution for 100*(1-α)% confidence with n degrees of freedom (n design settings). The prediction model can be rejected at 100*(1-α)% if the combination of multiple design points measured from physical experiments is outside of 100*(1-α)% confidence region.

According to Hills, a constant probability is given by the following ellipses where r^2 is constant for iso-probability curves.

$$r^2 = \begin{bmatrix} y_1 - y_{mean1} & y_2 - y_{mean2} & \cdots & y_n - y_{meann} \end{bmatrix} V^{-1} \begin{bmatrix} y_1 - y_{mean1} \\ y_2 - y_{mean2} \\ \cdots\cdots \\ y_n - y_{meann} \end{bmatrix} \tag{3}$$

In equation (3), y_i, i = *1...n*, stands for the single experimental result for each design setting *i*. y_{meani} is the mean of the random samples obtained from the computer model at each testing point *i*. The *V* matrix is the *n* by *n* co-variance matrix based on the random samples.

For model validation, the critical value of r^2 is obtained as:

$$r_{critical}{}^2 = l_{1-\alpha}^2(n)\,, \tag{4}$$

where *l* is the value associated with the 100*(1-α)% confidence for n testing points through the chi-square distribution. If the value of r^2 from equation (3) is less than the critical value of r^2 from equation (4), then we do not possess statistically significant evidence to declare our model invalid, and vice versa. When an acceptable error region is considered (see the box shown in Figure 3), the value of r^2 is calculated based on the location of the extreme corner of the box.

3.4 Data Transformations for Model Validation Purposes

As stated earlier, the strategies for model validation introduced in Hills and Trucano [1] are followed in this research. For comparison purposes, a test statistic r^2 is employed in this research. To apply the r^2 criterion for model validation, the assumption of normality of the multivariate joint probability distributions has to be satisfied.

Multivariate normality is the assumption that all dimensions and all combinations of the dimensions are normally distributed. When the assumption is met, the residuals (differences between predicted and obtained response values), are symmetrically distributed around a mean of zero and follows a normal distribution. The assumption of normality often leads to tests that are simple, mathematically tractable, and powerful compared to tests that do not make the normality assumption.

The two methods for normality screening are the statistical approach and the graphical approach. The statistical method employs examinations of significance for skewness and kurtosis. Mardia [39] suggested useful measures of skewness and kurtosis. Skewness is related to the symmetry of the distribution, while kurtosis is related to the peakedness of a distribution, either too peaked or too flat. The graphical method visually assesses the distributions of the data and compares them to the normal distribution.

Transformations can be applied to both the response (model output) and predictor (model input) variables following the approaches discussed in Section 2.4. Only after employing transformations can we apply the model validation procedure described earlier in Section 3. RSM, MCS and equations (3) and (4) are applied for the transformed model to assess the model validity.

3.5 Measurement error, response surface model error, and acceptable level of error

In the proposed model validation procedure, it is also important to consider various uncertainties (errors) that cannot be predicted by the uncertainty propagation based on the computational model. These errors include the measurement errors, the response surface model error, and the acceptable level of error. To simplify the process, we count the measurement errors and response surface model error by including them directly to the prediction uncertainty obtained through uncertainty propagation. Specifying an acceptable level of error is practically significant because the discrepancy between the simulated and experimental results indicates the errors associated with the model structure and numerical procedures (Types II and III uncertainties discussed in section 2). Approximated models should not be declared invalid if they provide predictions within an error that the user finds acceptable for a particular application. The acceptable level of error of a modeling approach is modeled as a box or a circle around the physical test point in Figure 3. Figure 3 shows a situation in which the confidence region of the model prediction and the acceptable error region overlap. This indicates that we cannot declare that the model is invalid for the given confidence level considering the acceptable level of error.

4. VALIDATING A FINITE-ELEMENT MODEL OF SHEET METAL FLANGING PROCESS

4.1 Sheet Metal Flanging Process and its Modeling

Sheet metal forming is one of the dominant processes in the manufacture of automobiles, aircraft, appliances, and many other products. As one of the most common processes for deforming sheet metals, flanging is used to bend an edge of a part to increase the stiffness of a sheet panel and (or) to create a mating surface for subsequent assemblies. As the tooling is retracted, the elastic strain energy stored in the material recovers to reach a new equilibrium and causes a geometric distortion due to elastic recovery (see Figure 4), the so-called “springback” [40]. Springback refers to the shape discrepancy between the fully loaded and unloaded configurations as shown in Figure 4.

Springback depends on a complex interaction between material properties, part geometry, die design, and processing parameters. The capability to model and simulate the springback phenomenon early in the new product design process can significantly reduce the product development cycle and costs. However, many factors influence the amount of springback in a physical test. Prediction and experimental testing of springback is particularly sensitive to the various types of uncertainties as discussed in [41], [42]. Referring to the definitions of the three types of uncertainties described in Section 2.1, examples of Type I uncertainty are the parameters related to incoming sheet metal material, initial geometry, and process setup. An example of Type II uncertainty is that, in material characterization, the hardening law to describe the behavior of sheet metal under loading and reverse loading is often uncertain; Example of Type III uncertainty is the numerical error caused by using different finite element analysis methods for spring back angle estimation, e.g., implicit Finite Element Method, explicit Finite Element Method, etc.

Insert Figure 4. Schematic of the springback in flanging

Various modeling approaches have been used to model the flanging process. These models include both analytical models and finite element analysis-based models. In this study, we illustrate how the proposed model validation approach can be applied to validate two finite element analysis models that model the blank plasticity with the combined hardening (Model 1) and isotropic hardening (Model 2) laws [43], respectively. The process is modeled by using an implicit and static nonlinear finite element code, ABAQUS/standard (v.5.8.). The two models with combined hardening and isotropic hardening laws are used to illustrate the effect of the fact when the data from MCS follow either normal or a non-normal distribution. Normalizing transformations are applied to the data from the model with the isotropic hardening law for model validation procedure. The angle at the fully unloaded configuration (see Figure 4) is considered as the process output (the response).

4.2 Problem Setup, Experiments, and Uncertainty Propagation in Validating Sheet Metal Forming Process Models

We illustrate in this section how the major phases in the proposed model validation approach are followed for our case study.

Phase I – Problem Setup

To accomplish Phase I, design variables and design parameters that affect the process output (final flange angle θ_f) are determined. Primarily, two design variables that are related to the process setup are considered, i.e. flange length, L; and gap space, g; and design parameters that are related to the material are selected, i.e. sheet thickness, t; and material properties (namely, Young's Modulus, E; Strain Hardening Coefficient, n; Material Strength Coefficient, K; and

Yield Stress, *Y*) (see Figure 5). Design parameters are uncontrollable (given) while design variables can be controlled over the design space to achieve the desired process output.

Insert Figure 5. System Diagram for Flanging Process

To form sample design settings, different combinations of values of design variables, i.e., *L* and *g*, are used. Five sample design settings are formed with combinations of low and high levels of flange length (3 and 5 inches) and gap (5 and 30 mm) plus a design point close to the middle (4 inches, 10 mm) (see Figure 6). These values for low, middle and high levels of flange length and gap are selected so that they can cover the whole design space as uniformly as possible.

Insert Figure 6. Sample Design Settings of Flanging Process for Model Validation

The variations of design variables and design parameters are identified in this phase. Based on experimental data, obtained by tensile tests, the relationships among *K*, *n*, and *Y* for carbon steel sheet metals used in the tests are approximated as

$$K(n) = 1128.5n + 499.97 \text{ , and} \tag{5}$$

$$Y(n) = -779.8n + 484.85 \text{ .} \tag{6}$$

Therefore, among the four parameters describing the material property, two are independent parameters (*n* and *E*), and the other two (*K* and *Y*) are dependent. Also, the statistical descriptions for these material parameters are obtained. The distribution of Young's Modulus (*E*) is assumed to be a normal distribution with 197949.7 MPa and 12914.7 MPa as the mean and standard deviation, respectively. The distribution of strain-hardening exponent (*n*) is assumed to be a uniform distribution from 0.10 to 0.18. The distributions of the strength coefficient (*K*) and yield stress (*Y*) depend on *n* as shown in equations 3 and 4. The distribution of sheet thickness (*t*) is assumed to be a normal distribution with 1.5529 mm and 0.0190 mm as the mean and standard deviation, respectively. Similarly, the variation of the design variable gap space (g) is assumed to be normally distributed with a standard deviation of 0.6 mm; note that the mean of the gap will change based on the location of the design point. The variation of flange length (L) is ignored as the flanging accuracy tolerance is insignificant comparing to the effect of the other change on final flange angle.

Phase II – Experiments and Measurements

In this phase, physical experiments are conducted and measurement errors are estimated. The dimensions of the sheet blank used in the physical experiments are 203.2 mm. x 203.2 mm. (or 8 inches x 8 inches). The flanging process uses a punch, a binder, a draw die, and a blank. The experiments have been performed by the 150-ton computer controlled HPM hydraulic press in the Advanced Materials Processing Laboratory at Northwestern University. The unloaded configurations, i.e., the angles between two planes in degrees (see Figure 4), have been measured by a coordinate measuring machine (Brown & Sharpe MicroVal Series Coordinate Measuring Machine B89) in the metrology laboratory at Northwestern University.

Phase III – Model Simulation and Uncertainty Propagation

The flanging process has been numerically simulated based on two finite element models, namely, one uses the combined hardening law (model 1) to the sheet material and model 2 uses the isotropic hardening law.

The process has been modeled by an implicit and static nonlinear commercial finite element code, ABAQUS/Standard. 1440 of eight-node, two-dimensional (plane strain) continuum elements with reduced integration have been used in this problem to model the sheet blank (ABAQUS element type CPE8R). The sheet thickness has been modeled with six layers. Tools have been modeled as rigid surfaces. The coefficient of friction is set to 0.125. The interface between the tooling and the sheet has been modeled by interface elements (IRS22) while the penalty-based contact algorithm has been used. To have a better convergence rate, the surface interaction is modeled by a soft contact. The analysis is performed in six steps: moving the binder toward the blank; developing the binder force; moving the punch down to flange the blank; retracting the punch up; releasing the binder force, and finally, moving the binder up.

The following two cases are considered in the simulation experiments for creating the response surface models for model validation. The procedure is illustrated here only with Model 1 (combined hardening law). A similar procedure is followed in validating Model 2 (isotropic hardening law).

Case 1: Validation at a single design point. 81 simulation experiments have been conducted to create a response surface model for model validation at a single design point (3, 30), i.e., flange length at 3 inches and gap at 30 mm. The response surface model represents the springback angle as a function across over a range of design parameters (*g*, *t*, *E*, *n*, *K*, *Y*) corresponding to a single design setting of L =3 inches. The 81 simulation experiments are designed based on various combinations of (*g, t, E, n, K, Y*), where three levels are considered for both gap (*g – 25, 30 and 35 inches at each level*) and thickness (*t – 1.483, 1.545 and 1.608 mm at each level*) and a full factorial design of these two factors are combined with nine settings of (*E*, *n*, *K*, *Y)* that capture a wide range of the material property.

Case 2: Validation for multiple design settings. 243 simulation experiments have been conducted to create the response surface model for model validation at five design points, i.e., the following combinations of design variables (flange length in inches, and gap in millimeters): (3, 5), (3, 30), (4, 10), (5, 30) and (5, 5). The response surface model represents the final flange angle as a function of (*L*, *g*, *t*, *E*, *n*, *K*, *Y*). The 243 simulation experiments are designed based on various combinations of (L, *g*, *t*, *E*, *n*, *K*, *Y*). Similar to the strategy used for designing the experiments in Case 2, three levels are considered for flange length (*L*), gap (*g*), and thickness (*t*) and a full factorial design of these three factors are combined with nine settings of (*E*, *n*, *K*, *Y)* that capture a wide range of the material property.

The second order Polynomial Regression (PR) approximation models are first used to create response surface models for both Cases 1 and 2. The accuracy is assessed by examining the sum of squares of error (SSE) based on a set of confirmation tests. For Model 1, the results are obtained as: SSE for PR is 0.0212 for Case 1 and SSE for PR is 0.9862 for 2. Considering that the magnitude of the angle of the final configuration is in the range of 100 to 150 degrees, the achieved SSE from PR is quite satisfactory. Therefore, for uncertainty propagation and model validation, the results from the polynomial models will be used.

Once the response surface models are created, the MCS has been used to efficiently predict the distributions of the final flange angle under uncertainty using 200,000 random sample points.

The uncertainty descriptions identified in Phase I, are followed for random sampling. The predicted distributions of the final flange angle will be presented together with the validity results next.

Phase IV – Model Validation via Comparisons

Normality Check

To simplify the model validation process, the predicted distributions of the final flange angle (for single design point and each individual design point in multiple design settings) have been checked for normality. The resulting probability distributions from Model 1 are plotted in Figure 7 (Case 1) and Figure 8 (Case 2).

Insert Figure 7. Confidence Limits based on Polynomial Model at Single Design Point

In Figures 7 and 8, the light pdf curve is the fitted normal distribution. It is noted that in general, the predictions (considered separately for each design point) based on polynomial models are all very close to normal. Kolmogorov-Smirnov (K-S) Test [44] has been conducted for normality check. Following the procedures in the literature, the sample size for K-S test is determined to be N=1000 (see [44], page 431). 1000 samples are randomly selected from the 200,000 simulations. If the K-S statistic obtained from a K-S test is greater than the critical value, here 0.043 for N=1000, the test rejects the Null Hypothesis (which states that the sample is drawn from a normal distribution). The K-S statistic is obtained as 0.04 for Case 1, which means we cannot reject the Null Hypothesis at α=0.05, and therefore the distribution can be considered as normal. The normality assumption can greatly simplify the validation process, which is introduced next.

Validation of Model 1 (Combined hardening law)

Case 1: For the single design point (3, 30), the results of the predicted springback angles based on MCS using the response surface model are compared with the result from a single physical experiment. As shown in Figure 7, the angle obtained from the experiment is 135.52 degrees, and 95.61% of the angles predicted with simulation based on the polynomial model, are smaller than the value of 135.52 (the left tail with the middle "Do not reject" area in Figure 7, together equal to 0.0439+0.9122=0.9561). Thus, [0.9561-(1-0.9561)]=0.9122 (the "Do not reject" area in Figure 7) is the confidence level with which one cannot reject the simulation model. The two tails (each equal to 1-0.9561=0.0439) are the “Reject the model” area.

Based on the identified critical confidence limit, we can say that if the confidence level is given at 90% (<91.22%), we can reject the model. If the confidence level is given at 95%, we cannot reject the model. We note that providing a higher confidence level, say 99%, would widen our acceptance region, while it will reduce our chances of rejecting a valid model, it would also increase our chance of accepting an invalid model.

Case 2: From the results of normality check conducted earlier, it is assumed that the total model uncertainty for five design points could be modeled by jointly distributed normal probability density functions. One physical experiment at each design point has been considered (see Figure 8), and the angles obtained from the experiments at each design point are the following: 134.9287, 106.5019, 111.6919, 135.2204, and 106.7697 at design points (3, 30), (3, 5), (4, 10), (5, 30), and (5, 5), respectively. Note that the physical experiments fall within the 95% confidence level at each design point. This means that the polynomial models considered separately at each individual design point cannot be rejected at the 95% confidence level.

Equations 3 and 4 have been used to calculate r^2 for the polynomial model. r^2 for the polynomial model is 7.4462 for Model 1. For the 95% confidence level, the critical value of r^2 is obtained as

$$r_{critical}{}^2 = l^2_{95\%}(5) = 11.07 \qquad (7)$$

Since the r^2 from the polynomial model is smaller than the critical r^2, there is not enough statistical evidence to conclude that the polynomial model is not valid. We find that for the polynomial model, the critical confidence limit (p-value) lies on about 80% contour because r^2 for 80%, for 5 dof = 7.289.

Insert Figure 8. Pdf Plots for Multiple Design Points, Model 1

<u>Validation of Model 2 (Isotropic hardening law).</u>

The validation of Model 2 is only illustrated for Case 2, i.e., for multiple design points, to demonstrate how data transformations can be applied to non-normal response distributions. The total model uncertainty for five design points again, has been modeled by jointly distributed normal probability density functions and one physical experiment at each design point has been considered. It is found that the response distributions obtained through the response surface models are non-normal at each design point. One can see in Figure 9 that the original distribution is right skewed with the right tail longer than the left tail. The null hypothesis for Kolmogorov-Smirnov test is rejected at α=0.05, and the KS statistic is 0.05 with P-value of P = 2.9408e-04. Plots of the probability density functions and the results of K-S tests at the other design points are similar to those provided for design point (3,30).

Insert Figure 9. Pdf Plot for Multiple Design Points, Model 2

Data transformations are applied to represent the distributions in scales that are close to normal. Unfortunately, the transformations (for response only) obtained by following the existing data transformation techniques (see Section 2.4) are not satisfactory because for this problem the ratio of the largest to the smallest observation is very close to one, a condition under which the existing techniques are not applicable. We then decide to apply transformations to both the response and the independent variables (model inputs) to overcome this difficulty. After some tests, it is found that when applying natural log transformations (λ=0) to both dependent (i.e., the angle at the unloaded configuration) and all independent variables (i.e., the design variables and parameters) from FEA model, the transformed distributions can be considered as normal. A polynomial response surface model is constructed using the transformed values of 243 observations from FEA at each design point (see Equation 8).

$$\ln Z = a + \sum_i b_i \ln X_i + \sum_{i,j} c_{ij} \ln x_i \ln x_j \tag{8}$$

These response surface models are used to predict the values of transformed dependent variables and to obtain their distributions at all design points. The SSE=0.00282179 for the new polynomial model in Equation 8 shows that the transformed model is quite accurate. The normality check and model validation for 200,000 sample points at each design setting are carried out for the obtained transformed model.

Figure 10 provides the pdf plot of 200,000 random samples at the design point (3,30) from the model corresponding to Equation 8. If we compare it with Figure 9, we can see that the plot has been improved in terms of normality, and the one in Figure 10 is close to a normal distribution. Kolmogorov-Smirnov Test is not rejected at α=0.05 with K-S statistic = 0.026. Plots of the probability density functions and the results of K-S tests for the other design points are similar to those provided for design point (3,30). The K-S statistics are 0.027, 0.029, 0.024 and 0.032 for the design points (3, 5), (4, 10), (5, 30) and (5, 5), respectively. Note that all of the values are smaller than the critical K-S statistic, i.e. 0.043. This indicates that the transformations to normal distributions are satisfactory.

Insert Figure 10. Transformed Pdf Plot, Model 2

With the transformed data, Equations 3 and 4 have been used to calculate r^2 for the transformed polynomial model 2, Case 2. r^2 for the original polynomial model 2 and for the transformed (both independent and dependent variables transformed) model 2 have been calculated to illustrate the effect of the transformation. Note that the natural logarithms of the angles from physical experiments are used in r^2 calculations. The r^2 for the transformed Model 2 is obtained as r^2 = 4.351, which is smaller than the critical r^2, 11.07 for 95% confidence level. Thus, for the specified confidence level, we cannot reject the polynomial isotropic hardening model. The critical confidence limit for the transformed data (i.e. for r^2 = 4.351) lies on about 52%. It is interesting to note that if we apply Equations 3 and 5 to the non-normal response distribution without applying data transformation, the original Model 2 is rejected as invalid for 95% confidence level. To make a statistically valid conclusion, data transformation needs to be applied.

We should note that we have not yet considered the errors of the response surface model, the experimental error, and the inaccuracy tolerance in the model validation process introduced so far. If considered, the modified confidence limit (*p* value) for rejecting a model is expected to be lower.

4.3 Measurement, Response Surface Model Errors, and Acceptable Level of Error

Following the statistical description in Section, the measurement error and the response surface model error are added directly to the predicted values of springback angle using MCS samples. The procedure is demonstrated here only for Model 1.

The mean and the variance of the response surface model error are estimated as the mean and the variance of the differences between the angles obtained from the FEM simulation and those predicted with the response surface model for 200 samples. The samples are obtained with Optimum Latin Hypercube Sampling (OLHS) [45]. As the result, the normal distribution N (-0.7120776, 0.605479) is used to describe the error of the polynomial model in Case 1, and N (-0.6311915, 1.0657299) is used to describe the error of the polynomial model in Case 2.

The mean and the variance of the measurement error are obtained based on the specification of the CMM machine, represented as N (0, 0.04193576). Simulation is used to obtain samples from normal distributions with the corresponding parameters as described above for the response surface model error and the measurement error.

The simulated errors are added observation-by-observation to the samples of springback angle from MCS for Case 1 and Case 2. Thus, new distributions that incorporate the errors are obtained for each case. The acceptable level of error is set to ±0.5 degree and incorporated by adjusting the result from the physical experiment. Thus, for the value of 135.52 (degrees) obtained from the physical experiment, the model cannot be rejected if the response distribution falls to the left of the minimum acceptable value of the experiment, i.e. 135.52 - 0.5 = 135.02 (degrees) (see Figure 11). In Figure 11, the curve with upper tails and lower pick reflects the modified pdf, which is checked against the lower limit of acceptable error range. Note that considering different errors reduces the possibility of rejecting a model.

Insert Figure 11. Considering Various Types of Errors

Case 1: Single design point: the confidence limit for 135.02 (degrees) is 87.95% for the polynomial model after the modification.

Case 2: Multiple design settings: r^2 for the polynomial model is 3.67 for the minimum acceptable values of the experiments for all design points, i.e., 134.4287, 106.0019, 111.1919, 134.7204, and 106.2697. As mentioned earlier, for the 95% confidence level, the critical value of r^2 is 11.07. Thus, the polynomial model for multiple design points cannot be rejected at 95% confidence level. The critical limit for the polynomial model lies on about 40% since r^2 for 40%, for 5 dof = 3.6555.

5. CONCLUSIONS

In this paper an approach for model validation via uncertainty propagation using the response surface methodology is presented. The approach uses response surface methodology to create metamodels as less costly approximations of simulation models for uncertainty propagation. Our proposed model validation procedure incorporates various types of uncertainties involved in a model validation process and significantly reduces the amount of physical experiments. The proposed approach can be used to provide stochastic assessment of model validity across a design space instead of a single point. The approach has been illustrated with an example of a sheet metal flanging process, for two finite element models (FEM) based on the combined hardening law and the isotropic hardening law, respectively. For the FEM model based on the combined hardening law, polynomial response surface models are created for both cases and confirmed to be accurate; they are used for uncertainty propagation in both cases. The critical confidence levels are identified by comparing the performance distribution obtained from uncertainty propagation with the results from the single experiments. The polynomial models have not been statistically declared as invalid if the given significant level is set at 95% for both single and multiple design points. The results are adjusted after considering the response surface model error, the measurement error, and the acceptable level of error.

For the FEM model based on the isotropic hardening law, the response distribution does not follow the normal distribution. The approach suggests to employ data transformations to the polynomial model based on isotropic hardening law. For the tested finite element model based on the combined hardening law, the model cannot be statistically declared as invalid if the given significant level is set at 95% for both single and multiple design points.

Future research will be directed toward integrating the model validation approach as a part of the model selection and decision making in engineering design.

Acknowledgments

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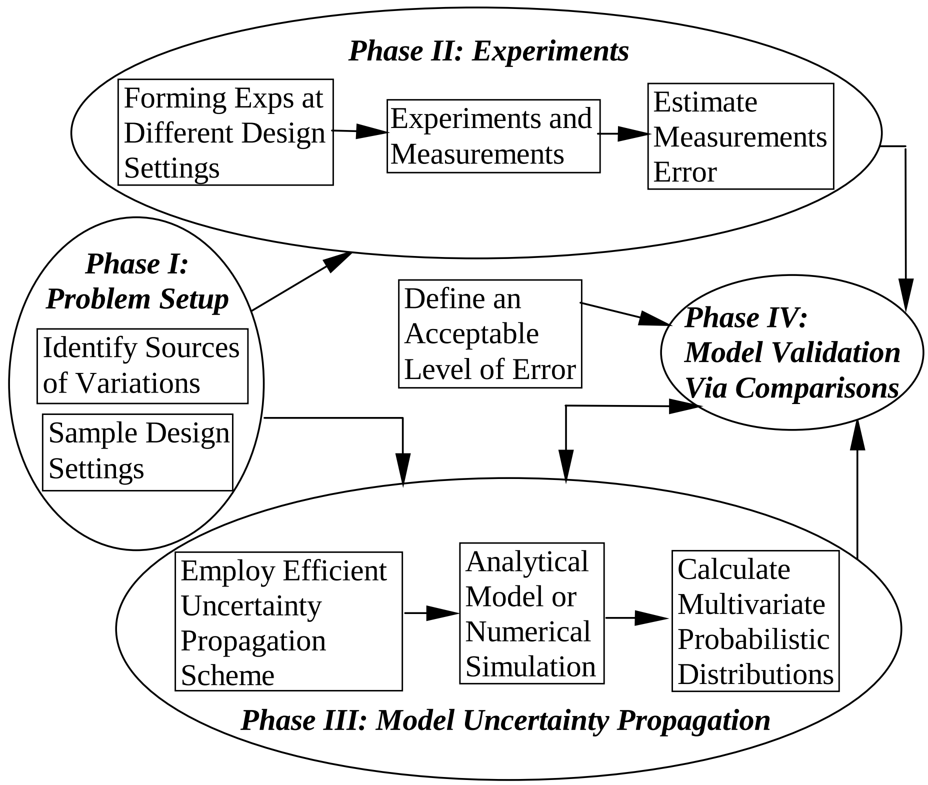


Figure 1. Procedure for Model Validation

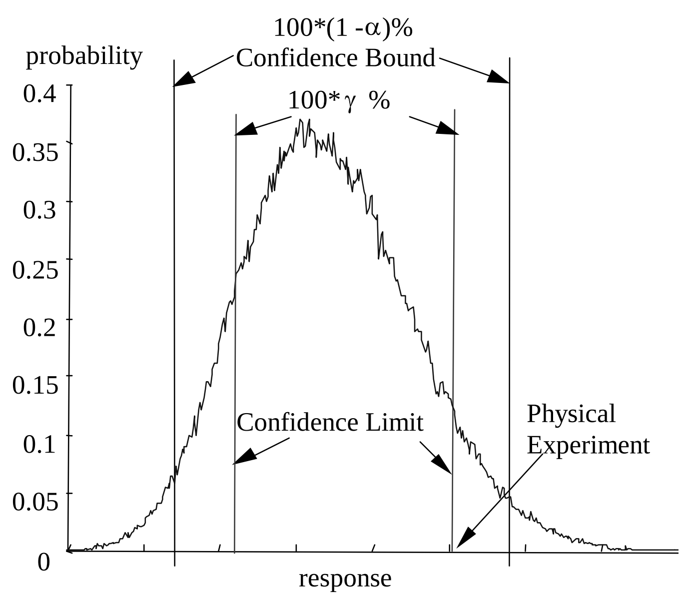


Figure 2. Model Validation for a Single Design Point

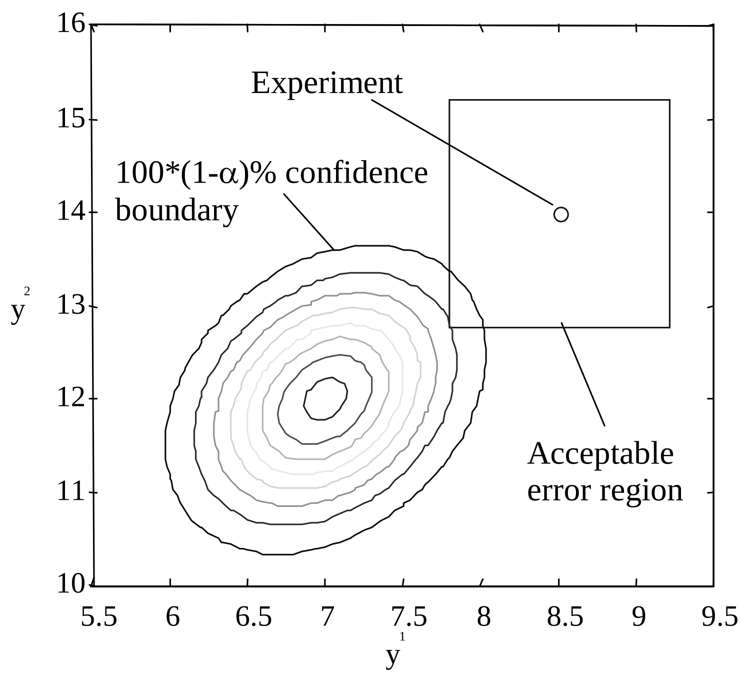


Figure 3. Model validation at two design settings

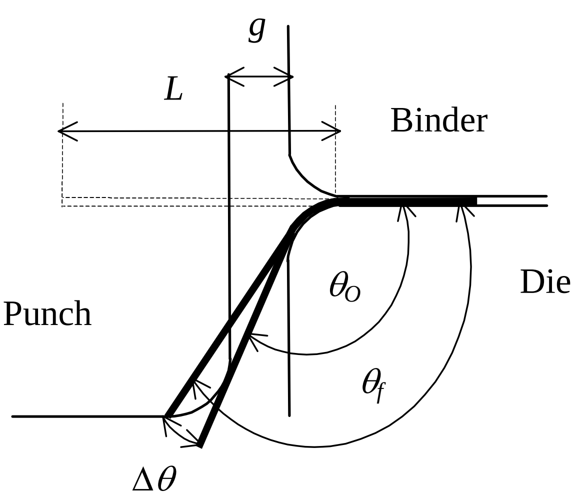


Figure 4. Schematic of the springback in flanging; *g* is the gap between the die and the punch, θ_0 is the flange angle at the fully loaded configuration, θ_f is that of the unloaded configuration, and $\Delta\theta$ is the springback.

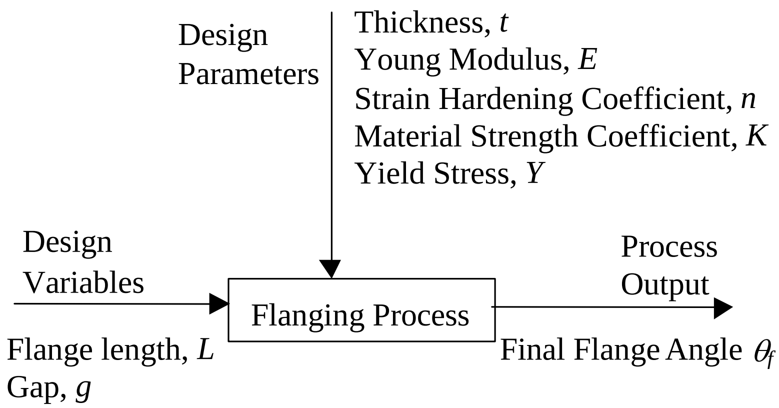


Figure 5. System Diagram for Flanging Process

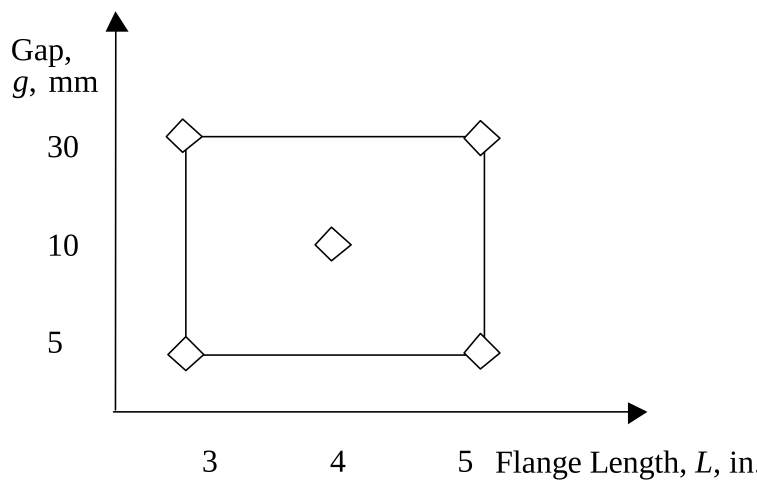


Figure 6. Sample Design Settings of Flanging Process for Model Validation

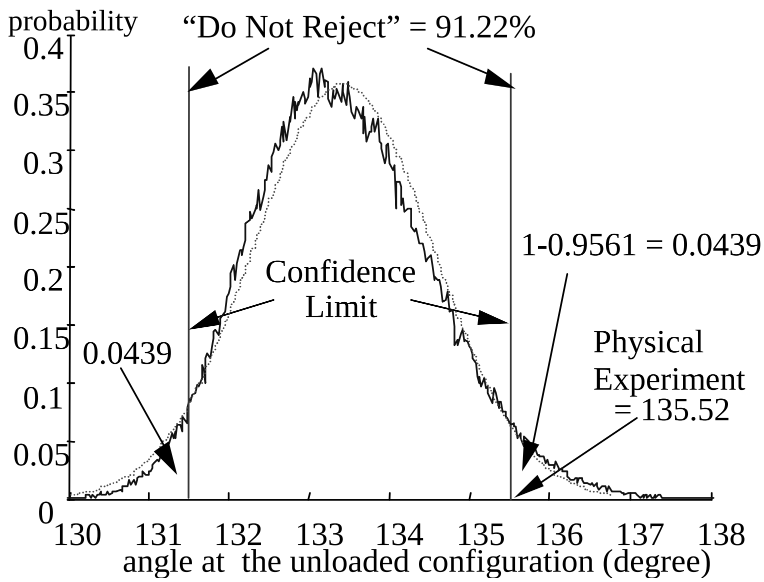
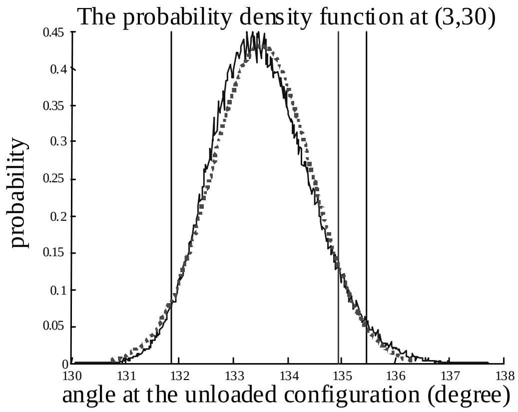
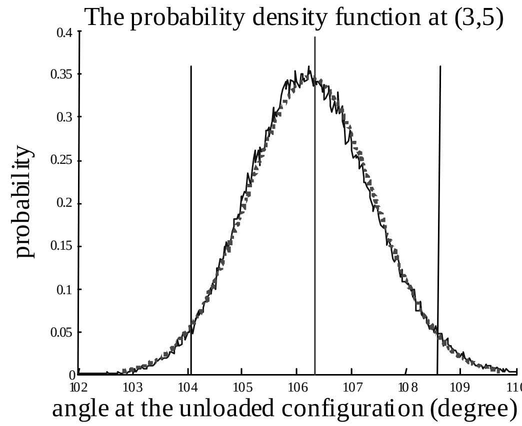


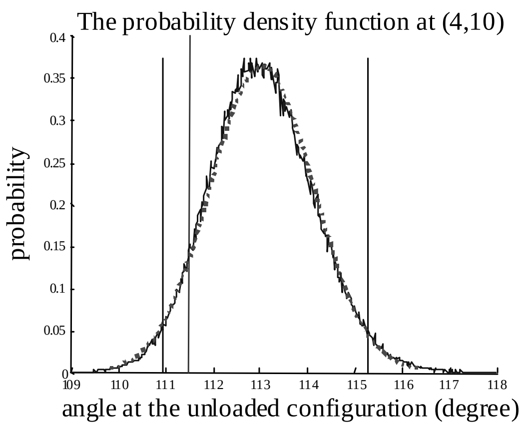
Figure 7. Confidence Limits based on Polynomial Model at Single Design Point (3, 30), Model 1. The Light curve is the pdf of fitted normal distribution



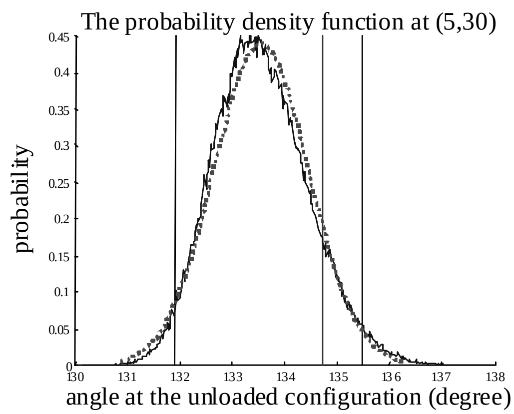
(a)



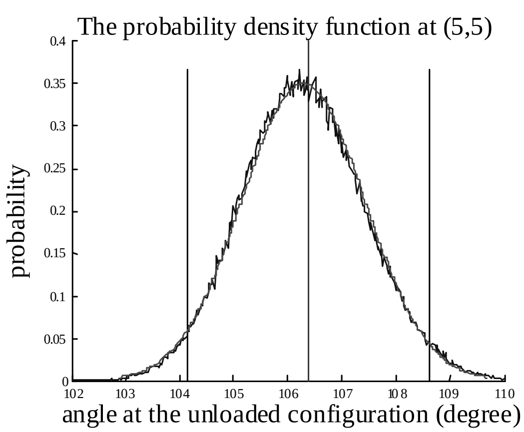
(b)



(c)



(d)



(e)

Figure 8. Pdf Plots for Multiple Design Points, Model 1; polynomial model at design points: (a) (3,30), (b) (3,5), (c) (4, 10), (d) (5,30), and (e) (5, 5). The light pdf curve is the fitted normal distribution at each design point. The two vertical lines are 95% confidence level and the line between them is the angle obtained from physical experiment at each design point.

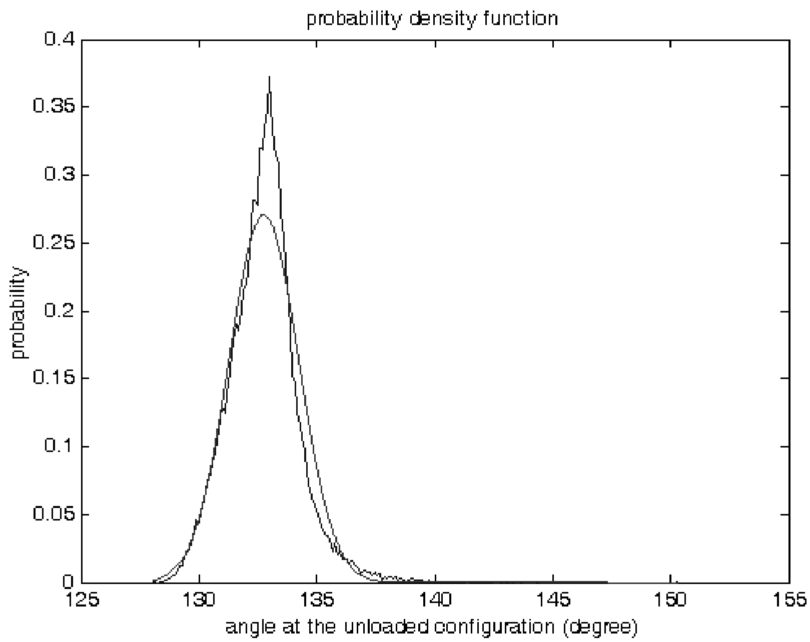


Figure 9. Pdf Plot for Multiple Design Points, Model 2; Polynomial model at design point (3,30). The light pdf curve is the fitted normal distribution.

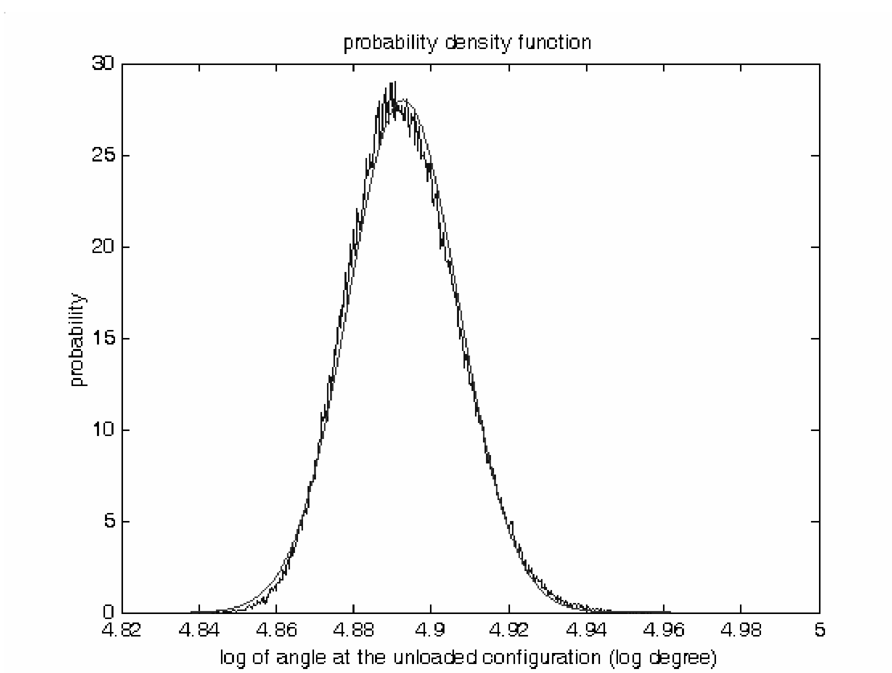


Figure 10. **Transformed Pdf Plot, Model 2;** after transforming the independent variable and the dependent variables, λ=0, at design point (3, 30). The light curve is the fitted normal distribution.

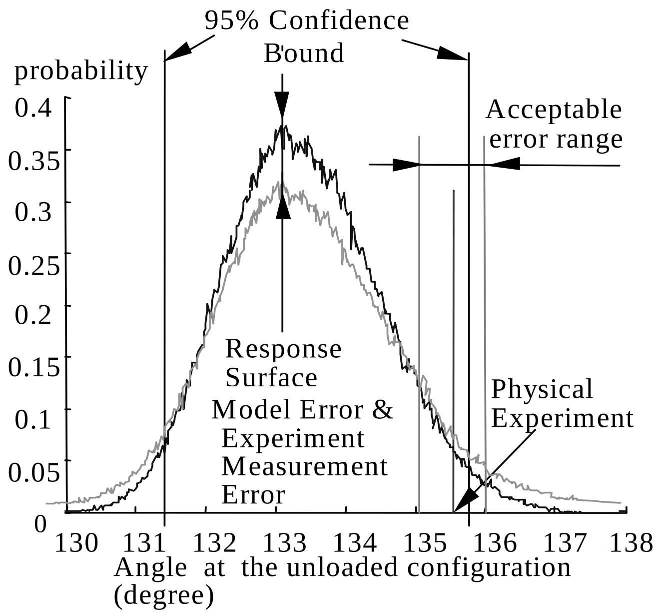


Figure 11. Considering Various Types of Errors