A Methodology for Managing the Effect of Uncertainty in Simulation-Based Design

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Simulation-based design has become an inherent part of multidisciplinary design as simulation tools provide designers with a flexible and computationally efficient means to explore the interrelationships among various disciplines. Complications arise when the simulation programs may have deviations associated with input parameters (*external uncertainties*) as well as *internal uncertainties* due to the inaccuracies of the simulation tools or system models. These uncertainties will have a great influence on design negotiations between various disciplines and may force designers to make conservative decisions. In this paper, an integrated methodology for propagating and mitigating the effect of uncertainties is proposed. Two approaches, namely, the *extreme condition approach* and the *statistical approach,* are developed to propagate the effect of uncertainties across a design system comprising interrelated subsystem analyses. Using the extreme condition approach, an interval of the output from a chain of simulations is obtained, while the statistical approach provides statistical estimates of the output. An uncertainty mitigation strategy based on the principles of robust design is proposed. The methodology is presented using an illustrative simulation chain and is verified using the case study of a six-link function-generator linkage design.

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Nomenclature

a	vector of system objective
c.d.f.	cumulative distribution function
f	response surface model (function)
F	vector of simulation function
g	vector of system constraint
p.d.f.	probability density function
RSM	response surface model
S	displacement of slider
w	weighting factor
x	vector of design variable
$\bar{\mathbf{x}}$	vector of nominal value of **x**
y	vector of linking variable
z	vector of system output
$\Delta\mathbf{x}$	vector of range of **x**
α	the maximum pressure angle
$\boldsymbol{\varepsilon}$	vector of error model
φ	crank angle
$\boldsymbol{\mu}$	vector of mean value
$\boldsymbol{\sigma}$	vector of standard deviation
ψ	rocker angle

I. Introduction

The advancements in Computer Aided Engineering (CAE) have resulted in the development of simulation tools that model the behavior of real world systems. These tools provide designers with flexible and cheap means to deal with complicated systems analysis and design under a multidisciplinary collaborative environment. Multidisciplinary systems design [1-4] usually involves interactions of various systems (called subsystems in this paper) connected by linking variables. These subsystems may be designed by different disciplines. Even though multidisciplinary optimization (MDO) has gained wide attention and applications, the treatment of uncertainties under multidisciplinary design has received very limited attention [5]. It is our aim in this paper to develop an integrated methodology for propagating and managing the effect of uncertainties in a simulation-based (multidisciplinary) systems design environment. In addition to discussing the various sources of uncertainties involved in simulation-based design, our focus is to illustrate the alternative techniques for propagating the effect of uncertainties across a design system comprising interrelated subsystem analyses, as well as to show the benefits of applying the robust design technique to making reliable design decisions under uncertainties.

It is generally recognized that there always exist uncertainties in any engineering systems due to variations in design conditions and mathematical models [6]. Omitting the algorithmic errors related to computer implementation, two general sources contribute to the uncertainties in simulation predictions:

- *External uncertainty*

External uncertainty comes from the variability in model prediction arising from plausible alternatives for input values (including both design parameters and design variables) [7,8]. It is also called "*input parameter uncertainty*". Examples include the variabilities associated with loading, material properties, physical dimensions of parts, and operating conditions.

- *Internal uncertainty*

  This type of uncertainty has two sources [9,10]. One is due to the limited information in estimating the characteristics of model parameters for a given, fixed model structure, which is called "*model parameter uncertainty*", and another type is in the model structure itself, including uncertainty in the validity of the assumptions underlying the model, referred to as "*model structure uncertainty*".

A critical issue in simulation-based systems design is that the effect of the uncertainties of one subsystem (or discipline) may propagate to another through linking variables and the final system output will have an accumulated effect of the individual uncertainties. A practical problem in large scale systems design is that multidisciplinary groups often use predictive tools of varying accuracy to determine if the design options are meeting the design requirements and to perform impact analyses of proposed changes from other groups. Some of these tools have good accuracy relative to test data (e.g., mechanical structural analysis). Others may have very low accuracy for engineering purposes (e.g., fatigue modeling). It is important to study the effect of various uncertainties as a part of requirements tracking and design coordination. Two primary issues that arise are

- How should the effect of uncertainties be propagated across the subsystems?

- How should we manage (mitigate) the effect of uncertainties and make reliable decisions?

Techniques for uncertainty analysis exist widely in the literature. The extreme condition approach (or worst case analysis) and the statistical approach are the two commonly used approaches. The common extreme condition approach is to derive the range of system output in terms of the range of uncertainties by either sub-optimizations [11], first-order Taylor expansion, or interval analysis[12]. The statistical approach relies heavily on the use of data sampling to generate cumulative distribution function (*c.d.f.*) of system outputs. Monte Carlo simulation [13], a random simulation based approach could become very expensive in some of the applications. Reduced sampling techniques, like Latin Hypercube Sampling [14], importance sampling [15], and Taguchi's orthogonal arrays [16], may be used to improve the computational efficiency. In most of the existing applications, the use of extreme condition approach and the statistical approach has been restricted to propagating the effect of external uncertainty but not that of the internal uncertainty, let alone the combination of both. In recent developments, some preliminary results of propagating the effect of model uncertainty (internal uncertainty) are reported[17]. In their work , model uncertainty is denoted by a range (bias) of the system output. With this simplistic treatment, the "worst case" concept and the first-order sensitivity analysis are used to evaluate the deviations of an end performance. There is a need to accommodate more generic representations of both external and internal uncertainties. In the content of optimization, the physical programming method[18,19] also uses ranges to express performance preference for each objective. This approach may also entail inherent robustness properties.

There are few works associated with how to mitigate the effect of both the external and internal uncertainties in simulation-based design. While in the past a lot of efforts were spent on reducing the magnitude of variation sources, recent development in design techniques has generated methods that can reduce the impact of potential variations by manipulating controllable design variables. Taguchi's Robust Design is such an approach that emphasizes reduction of performance variation through reducing sensitivity to sources of variation [16]. A part of authors' work has been on developing nonlinear programming methods that can be used for a variety of robust design applications, as well as overcoming the mathematical limitations of the methods Taguchi offered [6, 20, 21]. Robust design has also been used at the system level to reduce the performance variation caused by manufacturing deviations[22]. In this work, the concept of robust design is used to mitigate performance variations due to various sources of uncertainties in simulation-based design. In this paper, an integrated methodology for the propagating and managing the effect of uncertainties is proposed. Two approaches, namely, the *extreme condition approach* and the *statistical approach,* are developed to propagate the effect of both the external uncertainty and the internal uncertainty across a design system comprising interrelated subsystem analyses. An uncertainty mitigation strategy based on the principles of robust design is proposed. A simplistic simulation chain model is used to explain the proposed methodology and a six-link function-generator linkage design problem is used to illustrate the benefits of applying the robust design approach for uncertainty mitigation. The principles of the proposed methods can be easily extended to more complicated, real multidisciplinary design problems.

II. Propagation of the Effect of Uncertainties

In this section, a simulation-based design model as illustrated in Fig. 1 is used to explain the proposed methodology. The model consists of a chain of two simulation programs (imagining they are from two different disciplines) that are connected to each other through *linking variables* represented by the vector $\mathbf{y}$. The input to simulation model I is the vector of design variables — $\mathbf{x}_1$ with uncertainty (*external uncertainties*, describe by a range $\Delta\mathbf{x}_1$ or certain distributions).

Insert Fig. 1 here.

For simulation model I, the output $\mathbf{y}$ can be expressed as

$$\mathbf{y} = \mathbf{F}_1(\mathbf{x}_1) + \boldsymbol{\varepsilon}_1(\mathbf{x}_1), \tag{1}$$

where $\mathbf{F}_1(\mathbf{x}_1)$ is the simulation model and $\boldsymbol{\varepsilon}_1(\mathbf{x}_1)$ is the corresponding error model of the *internal uncertainty*. Additive error model is used to represent model structure uncertainty in this study, though its real form can be much more complicated.

For simulation model II, the inputs are the linking variable $\mathbf{y}$ and the design variable $\mathbf{x}_2$. The output vector $\mathbf{z}$ can be expressed as

$$\mathbf{z} = \mathbf{F}_2(\mathbf{x}_2, \mathbf{y}) + \boldsymbol{\varepsilon}_2(\mathbf{x}_2, \mathbf{y}), \tag{2}$$

where $\mathbf{F}_2(\mathbf{x}_2, \mathbf{y})$ is the simulation model and $\boldsymbol{\varepsilon}_2(\mathbf{x}_2, \mathbf{y})$ is the corresponding error model. The output $\mathbf{z}$ often represents system performance parameters that are used to model the design objectives and constraints. Because of the deviations existing in $\mathbf{x}_2$ and $\mathbf{y}$, and the *internal uncertainty* $\varepsilon_2(\mathbf{x}_2, y)$, the final output $\mathbf{z}$ will also have deviations.

The question is how to propagate the effect of various types of uncertainties across a simulation chain with interrelated simulation programs. Two approaches, the *extreme condition approach* and the *statistical approach,* are presented in the following sections.

A. Extreme Condition Approach for Uncertainty Analysis

The *extreme condition approach* is developed to obtain an interval or the *extremes* of the final output from a chain of simulation models. The term *extreme* is defined as the minimum or the maximum value of the end performance (final output) corresponding to the given ranges of internal and external uncertainties. With this approach, the external uncertainties are characterized by the intervals $[\bar{\mathbf{x}}_1 - \Delta\mathbf{x}_1, \bar{\mathbf{x}}_1 + \Delta\mathbf{x}_1]$ and $[\bar{\mathbf{x}}_2 - \Delta\mathbf{x}_2, \bar{\mathbf{x}}_2 + \Delta\mathbf{x}_2]$ ($\bar{\mathbf{x}}_1$ and $\bar{\mathbf{x}}_2$ denote the nominal values of $\mathbf{x}_1$ and $\mathbf{x}_2$ respectively). Correspondingly, the outputs of the two simulation models are described by the intervals $[\mathbf{y}^{\min}, \mathbf{y}^{\max}]$ and $[\mathbf{z}^{\min}, \mathbf{z}^{\max}]$, respectively.

Optimizations are used to find the maximum and minimum (extremes) of the outputs from simulation model I and simulation model II, respectively. The flow chart of the proposed procedure is illustrated in Fig. 2. The steps to obtain the range of output **z**, $[\mathbf{z}^{\min}, \mathbf{z}^{\max}]$, are presented as:

a) Given a set of nominal values $\bar{\mathbf{x}}_1$, $\bar{\mathbf{x}}_2$ and ranges $\Delta\mathbf{x}_1$, $\Delta\mathbf{x}_2$;

b) For simulation model I, minimize and maximize $\mathbf{F}_1(\mathbf{x}_1)$ over the range of $[\bar{\mathbf{x}}_1 - \Delta\mathbf{x}_1, \bar{\mathbf{x}}_1 + \Delta\mathbf{x}_1]$ to obtain $\mathbf{F}_1^{\min}(\mathbf{x}_1)$ and $\mathbf{F}_1^{\max}(\mathbf{x}_1)$[#]. The optimization model is

Given:	The nominal value of $\bar{\mathbf{x}}_1$ and the range $\Delta\mathbf{x}_1$	(3)
Subject to:	$\bar{\mathbf{x}}_1 - \Delta\mathbf{x}_1 \le \mathbf{x}_1 \le \bar{\mathbf{x}}_1 + \Delta\mathbf{x}_1$	
Optimize:	Minimize $\mathbf{F}_1(\mathbf{x}_1)$ to obtain $\mathbf{F}_1^{\min}(\mathbf{x}_1)$	

[#] In this paper, though vector representation is used for simplicity, the objective functions in optimization are scalar valued.

Maximize $\mathbf{F}_1(\mathbf{x}_1)$ to obtain $\mathbf{F}_1^{\max}(\mathbf{x}_1)$

c) Similar to step b), obtain the extreme values of internal uncertainty $\boldsymbol{\varepsilon}_1^{\min}(\mathbf{x}_1)$ and $\boldsymbol{\varepsilon}_1^{\max}(\mathbf{x}_1)$ over the range of $[\mathbf{x}_1 - \Delta\mathbf{x}_1, \mathbf{x}_1 + \Delta\mathbf{x}_1]$;

d) Obtain the interval $[\mathbf{y}^{\min}, \mathbf{y}^{\max}]$ using

$$\mathbf{y}^{\min} = \mathbf{F}_1^{\min}(\mathbf{x}_1) + \boldsymbol{\varepsilon}_1^{\min}(\mathbf{x}_1) \text{ and} \tag{4}$$

$$\mathbf{y}^{\max} = \mathbf{F}_1^{\max}(\mathbf{x}_1) + \boldsymbol{\varepsilon}_1^{\max}(\mathbf{x}_1) \tag{5}$$

e) For simulation model II, minimize and maximize $\mathbf{F}_2(\mathbf{x}_2, \mathbf{y})$ over the range of $[\bar{\mathbf{x}}_2 - \Delta\mathbf{x}_2, \bar{\mathbf{x}}_2 + \Delta\mathbf{x}_2]$ and $[\mathbf{y}^{\min}, \mathbf{y}^{\max}]$ to obtain $\mathbf{F}_2^{\min}(\mathbf{x}_2, \mathbf{y})$ and $\mathbf{F}_2^{\max}(\mathbf{x}_2, \mathbf{y})$. The optimization model is similar to the one in step b).

f) Similar to step e), obtain the extreme values of internal uncertainty $\boldsymbol{\varepsilon}_2^{\min}(\mathbf{x}_2, \mathbf{y})$ and $\boldsymbol{\varepsilon}_2^{\max}(\mathbf{x}_2, \mathbf{y})$;

g) Obtain the interval $[\mathbf{z}^{\min}, \mathbf{z}^{\max}]$ using

$$\mathbf{z}^{\min} = \mathbf{F}_2^{\min}(\mathbf{x}_2, \mathbf{y}) + \boldsymbol{\varepsilon}_2^{\min}(\mathbf{x}_2, \mathbf{y}) \text{ and} \tag{6}$$

$$\mathbf{z}^{\max} = \mathbf{F}_2^{\max}(\mathbf{x}_2, \mathbf{y}) + \boldsymbol{\varepsilon}_2^{\max}(\mathbf{x}_2, \mathbf{y}) \tag{7}$$

Based on the computed interval $[\mathbf{z}^{\min}, \mathbf{z}^{\max}]$, the nominal value of $\mathbf{z}$ is calculated as

$$\bar{\mathbf{z}} = \frac{1}{2}(\mathbf{z}^{\min} + \mathbf{z}^{\max}) \tag{8}$$

The deviation of $\mathbf{z}$ can be calculated as

$$\Delta\mathbf{z} = \mathbf{z}^{\max} - \mathbf{z}^{\min} \tag{9}$$

Insert Fig. 2 here.

The extreme condition approach identifies the interval of a system output based on the given intervals of the system inputs. It is applicable to the situation in which both the external uncertainties in $\mathbf{x}_1$ and $\mathbf{x}_2$ are expressed by ranges.

B. The Statistical Approach for Uncertainty Analysis

The statistical approach is developed to estimate *c.d.f.*, probability density functions (*p.d.f.*) or population parameters (for example, mean and variance) of the final outputs from a chain of simulation models. Here we assume $\mathbf{x}_1$, $\mathbf{x}_2$, and the internal uncertainty $\boldsymbol{\varepsilon}_1(\mathbf{x}_1)$ and $\boldsymbol{\varepsilon}_2(\mathbf{x}_2, \mathbf{y})$ follow certain probabilistic distributions that may be obtained by field or experimental data, the information of similar existing products, and judgements by engineering experience. Note, since the distribution parameters (for example, mean and variance) of $\boldsymbol{\varepsilon}_1(\mathbf{x}_1)$ and $\boldsymbol{\varepsilon}_2(\mathbf{x}_2, \mathbf{y})$ are functions of $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{y}$, the final distributions of $\boldsymbol{\varepsilon}_1(\mathbf{x}_1)$ and $\boldsymbol{\varepsilon}_2(\mathbf{x}_2, \mathbf{y})$ are the accumulated effects of both the uncertainty in the error model and the uncertainty of the external parameters such as $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{y}$. Monte Carlo simulation methods [23] are used to propagate the effect of uncertainties through the simulation chain. The procedure is developed as:

a) Generate H samples of $\mathbf{x}_1$ and $\mathbf{x}_2$ as simulation inputs based on their distribution functions;

b) For the given $\mathbf{x}_1$, calculate the distribution parameters of the internal uncertainty $\boldsymbol{\varepsilon}_1(\mathbf{x}_1)$ for simulation model I and generate N samples of the internal uncertainty $\boldsymbol{\varepsilon}_1$ for simulation model I based on the distribution function;

c) Evaluate the corresponding output $\mathbf{y} = \mathbf{F}_1(\mathbf{x}_1) + \boldsymbol{\varepsilon}_1$ for simulation model I;

d) For each $\mathbf{y}$, calculate the distribution parameters of the internal uncertainty $\boldsymbol{\varepsilon}_2(\mathbf{x}_2, \mathbf{y})$ of simulation model II and generate M samples of the internal uncertainty $\boldsymbol{\varepsilon}_2$ based on the distribution function;

e) Evaluate the corresponding output $\mathbf{z} = \mathbf{F}_2(\mathbf{x}_2, \mathbf{y}) + \boldsymbol{\varepsilon}_2$ for simulation model II;

f) Calculate the mean value $\boldsymbol{\mu}_z$, the standard deviation $\boldsymbol{\sigma}_z$, or the *c.d.f.* and *p.d.f.* of $\mathbf{z}$ based on $\mathrm{H} \times \mathrm{M} \times \mathrm{N}$ samples of $\mathbf{z}$.

Insert Fig. 3 here.

Figure 3 depicts the process of this Monte Carlo simulation-based approach. This approach generates statistical estimates of the system output based on the given distributions of the inputs and error models. This gives us more information than the extreme condition approach by which only the best and worst performance are estimated. Because the statistical approach is based on the concept of Monte Carlo simulation, it often requires a large number of simulations. More effective sampling techniques such as Latin Hyper Cube[24] and fractional factorial design[25] can be used to reduce the amount of simulations.

III. Mitigating the effect of Uncertainty

To assist designers to make reliable design decisions under uncertainties, we integrate the proposed techniques of propagating the effect of uncertainties with the multidisciplinary optimization approach based on the principles of robust design, i.e. to extend the quality engineering concept to the mitigation of the effects of both external and internal uncertainties. From the viewpoint of robust design[16, 26], the goal is to make the system (or product) least sensitive to the potential variations without eliminating the sources of uncertainty. The same concept is used here to reduce the impact of both external and internal uncertainties associated with the simulation programs. The robust optimization objective is achieved by simultaneously "optimizing the mean performance" and "reducing the performance variation", subject to the constraints [6,20] considering their deviations. Note Taguchi's robust design has been used in the past for mitigating the effect of parameter uncertainty which is similar to the external uncertainty considered here. In this work, the concept is extended to mitigate the effect of model structure uncertainty in a similar manner.

For the extreme condition approach, the robust design model can be formulated as:

Given:	Parameter and model uncertainties (ranges)
Find:	Robust design decisions ($\mathbf{x}$)
Subject to:	System Constraints:
	$\mathbf{g}_{worst}(\mathbf{x}) \leq 0$ (10)
Objectives:	a. Optimize the mean of system attributes: $\bar{\mathbf{a}}(\mathbf{x})$
	b. Minimize the deviation of system attributes: $\Delta\mathbf{a}(\mathbf{x})$

In the above model, $\mathbf{g}_{worst}(\mathbf{x})$ is the maximum constraint function estimated by the worst case of constraint function $\mathbf{g}(\mathbf{x})$ and $\mathbf{a}$ is the objective vector. Both $\mathbf{g}(\mathbf{x})$ and $\mathbf{a}(\mathbf{x})$ are the subsets of system output vector $\mathbf{z}$. The mean and deviation of the system outputs can be obtained by the extreme condition approach as introduced earlier. Note that we have multiple objectives in robust design, i.e., both the mean and the deviation of the system are expected to be minimized (here we assume optimizing the mean of a system attribute can always be transformed into a minimization problem). The general form of the objective can be expressed as

$$\min \quad [\bar{\mathbf{a}}(\mathbf{x}), \Delta\mathbf{a}(\mathbf{x})] \tag{11}$$

Many existing approaches can be used to solve the above multiobjective robust optimization problem [27-30].

In the above model, we use the worst case analysis [27] to formulate the constraints (Eqn. 10). The worst case analysis assumes that all fluctuations may occur simultaneously in the worst possible combination. The effect of variations on a function is estimated using a first order Taylor's series as follows:

$$\Delta \mathbf{g}(\mathbf{x}) = \sum_i \left| \frac{\partial \mathbf{g}(\mathbf{x})}{\partial x_i} \Delta \mathbf{x} \right|, \tag{12}$$

where $\Delta \mathbf{g}(\mathbf{x})$ represents the variation transmitted to constraint $\mathbf{g}(\mathbf{x})$ for a worst case analysis. Then the design feasibility in Eqn. 10 can be formulated by increasing the value of the mean $\mathbf{g}(\bar{\mathbf{x}})$ by the amount of functional variation $\Delta \mathbf{g}(\mathbf{x})$, i.e.,

$$\mathbf{g}_{worst}(\mathbf{x}) = \mathbf{g}(\bar{\mathbf{x}}) + \sum_i \left| \frac{\partial \mathbf{g}(\mathbf{x})}{\partial x_i} \Delta x_i \right| \tag{13}$$

When using the statistical approach to estimate the performance distribution, the robust model can be formulated as:

Given: Parameter and model uncertainties (distributions)
Find: Robust design decisions $\mathbf{x}$
Subject to: System Constraints:
$P[\mathbf{g}(\mathbf{x}) \leq 0] \geq P_{\text{limit}}$ (14)
Objectives: a. Optimize the mean of system attributes $\mathbf{a}(\mathbf{x})$: $\boldsymbol{\mu}_{\mathbf{a}}(\mathbf{x})$
b. Minimize the standard deviation of system attributes $\mathbf{a}(\mathbf{x})$: $\boldsymbol{\sigma}_{\mathbf{a}}(\mathbf{x})$

$\boldsymbol{\mu}_{\mathbf{a}}(\mathbf{x})$ and $\boldsymbol{\sigma}_{\mathbf{a}}(\mathbf{x})$ are the estimates of mean and variance of the system outputs respectively. Note that the constraints in the above model are expressed by the probabilistic formulation. $P[\mathbf{g}(\mathbf{x}) \leq 0]$ is the probability of constraint satisfaction and it should be bigger than or equal to the defined probability limit P_{limit}. Because it is very computationally expensive to evaluate the probability of constraint satisfaction, alternative formulations, for example, the moment matching method [6, 27, 28], are used in practice to evaluate the constraints.

With the moment matching method, if $\mathbf{g}(\mathbf{x})$ is assumed to follow a normal distribution. The constraint in Eqn. 14 is formulated as

$$\boldsymbol{\mu}_{\mathrm{g}} + k\boldsymbol{\sigma}_{\mathrm{g}} \leq 0\,, \tag{15}$$

where k is a constant which stands for the probability of constraint satisfaction [27]. For example, $k = 1$ stands for the probability ≈ 0.8413 and $k = 2$ means the probability ≈ 0.9772.

Based on the previous discussions, the strategy that integrates the propagation and mitigation of the effect of uncertainties is summarized in Figure 4. There are three modules in the integrated method. Module A is the *uncertainty quantification module*. This module represents the first stage in the integrated methodology. Module B is the *propagation module*. In this module, either the extreme condition approach or the statistical approach introduced earlier is used to identify the range or to estimate the population parameters of system performance under the influence of both internal and external uncertainties. The obtained final performance ranges or estimated population parameters are then used in the Module C, the *management module*, to mitigate the effect of uncertainties. The purpose of this module C is to obtain the values of design variables that are tolerant to the uncertainties. The basis for controlling the effect of uncertainties is the robust design approach. The formulation can be found in Eqns. 10 and 14. It should be noted that the process to manage the effect of uncertainty is iterative and involves repeated uncertainty analysis until a robust optimal solution is obtained.

Insert Fig. 4 here.

IV. Example—Mechanism Synthesis

The design of a six-link function-generator linkage (See Fig. 5) is used as a case study to illustrate the tangible effects of the proposed approach. The functional requirements of the mechanism are that when the input angle φ (crank angle) varies between 30° and 60°, the output displacement (slider displacement) S is desired to follow the function:

$$S = 2.17436 + 0.02016\varphi - 0.00039\varphi^2 + 1.73\times10^{-6}\varphi^3 \quad (16)$$

The length of crank AB, the length of frame AD*,* and the angle β are given as $AB = 0.85\,mm$, $AD = 1.9\,mm$, and $\beta = 0^\circ$. The maximum pressure angle α_1 of the four-bar linkage ABCD and the maximum pressure angle α_2 of the four-bar linkage DEF must be less than 55° and 26°, respectively. The design variables are $\mathbf{x} = [x_1, x_2, x_3, x_4] = [BC, CD, DE, EF]$.

Insert Fig.5 here.

For the purpose of illustration, we decompose the given system into two subsystems, the first four-bar mechanism ABCD and the second four-bar mechanism DEF, which are shown in Figs. 6 and 7, respectively. The two subsystem analyses are considered as simulations I and II as defined earlier in Fig. 1.

Insert Fig.6 here.

Insert Fig.7 here.

In terms of the inputs and outputs of the two simulation programs, for subsystem I, the inputs are φ, x_1 and x_2, and the outputs are the angle (ψ) of rocker CD and the maximum pressure angle α_1. For subsystem II, the inputs are ψ, x_3 and x_4, and the outputs are the displacement of slider S and the maximum pressure angle α_2.

The analytical models to design the mechanism can be found in references[31, 32]. Rather than using the analytical models directly, we create response surface models (RSMs) as the simulation models for these two subsystems and use them to design the mechanism based on the proposed methodology. The purpose of creating the RSMs in this study is not to improve the computational efficiency through approximations as the way they are normally used. Rather the purpose is to illustrate how to mitigate the effect of uncertainty when simplified models are used in design. The errors introduced by RSMs are considered explicitly as the uncertainty associated with the model structure. The benefits of the proposed method are illustrated by comparing the results from RSMs, both with and without the consideration of uncertainty, to those from using real analytical models.

For subsystem I, the RSMs created are $\psi = f_{\psi}(\varphi, x_1, x_2)$ and $\alpha_1 = f_{\alpha_1}(x_1, x_2)$. These models are considered as the simulation models $\mathbf{F}_1(\mathbf{x}_1)$ as defined in Fig. 1,

$$\mathbf{F}_1(\mathbf{x}_1) = \mathbf{F}_1(x_1, x_2) = [f_{\psi}(\varphi, x_1, x_2), f_{\alpha_1}(x_1, x_2)] \tag{17}$$

For subsystem II, the RSMs are $S = f_S(\psi, x_3, x_4)$ and $\alpha_2 = f_{\alpha_2}(x_3, x_4)$. These models are considered as the simulation models $\mathbf{F}_2(\mathbf{x}_2, \mathbf{y})$,

$$\mathbf{F}_2(\mathbf{x}_2, \mathbf{y}) = \mathbf{F}_2(x_3, x_4, \psi) = [f_S(\psi, x_3, x_4), f_{\alpha_2}(x_3, x_4)] \tag{18}$$

Different combinations of design variables (x_1, x_2, x_3, x_4) and input angle φ are selected for computer simulations using the analytical models. Based on the simulation results, a standard multilinear regression is used to fit second-order response surfaces models $f_{\psi}(\varphi, x_1, x_2)$, $f_{\alpha_1}(x_1, x_2)$, $f_S(\psi, x_3, x_4)$, and $f_{\alpha_2}(x_3, x_4)$ in the following form

$$y = a_0 + \sum_{i=1}^{m} a_i x_i + \sum_{i=1}^{m} a_{ii} x_i^2 \ , \tag{19}$$

where y is the approximate response, m is the number of input variables, a_i and a_{ii} are regression coefficients.

The error models corresponding to the RSMs are $\boldsymbol{\varepsilon}_1(\mathbf{x}_1) = [\varepsilon_\psi(\varphi, x_1, x_2), \varepsilon_{\alpha_1}(x_1, x_2)]$ and $\boldsymbol{\varepsilon}_2(\mathbf{x}_2, \mathbf{y}) = [\varepsilon_S(\psi, x_3, x_4), \varepsilon_{\alpha_2}(x_3, x_4)]$. With the statistical approach, for simplicity, we assume all the errors can be modeled using normal distributions. The mean values and standard deviations of the errors are denoted by μ_ψ, σ_ψ, μ_{α_1}, σ_{α_1}, μ_S, σ_S, μ_{α_2} and σ_{α_2} respectively (μ represents mean value and σ represents standard deviation). The probability density function of the error models has the following form:

$$p.d.f.\ (\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2} (\frac{\varepsilon - \mu}{\sigma})^2 \right] \tag{20}$$

The parameters μ_ψ, σ_ψ, μ_{α_1}, σ_{α_1}, μ_S, σ_S, μ_{α_2} and σ_{α_2} are estimated using 100 samples of errors, which are evaluated by the differences between the values from the analytical models and those from RSMs. The samples are randomly picked over the range $30^\circ \le \varphi \le 60^\circ$.

For the extreme condition approach, we specify the internal uncertainties as $\varepsilon_\psi \in [\mu_\psi - 3\sigma_\psi, \mu_\psi + 3\sigma_\psi]$, $\varepsilon_{\alpha_1} \in [\mu_{\alpha_1} - 3\sigma_{\alpha_1}, \mu_{\alpha_1} + 3\sigma_{\alpha_1}]$, $\varepsilon_S \in [\mu_S - 3\sigma_S, \mu_S + 3\sigma_S]$ and $\varepsilon_{\alpha_2} \in [\mu_{\alpha_2} - 3\sigma_{\alpha_2}, \mu_{\alpha_2} + 3\sigma_{\alpha_2}]$.

Following the structure of the simulation chain defined in Fig.1, Fig. 8 particularizes the relationship of the sub-simulation programs for the six-link linkage design problem.

Insert Fig. 8 here.

The weighted sum method is used in our study to model the multiple objectives in robust design. For the statistical approach, the robust optimization model is stated as

$$Min\ F(x_1, x_2, x_3, x_4) = w_1 \sum_{i=1}^{n} [\mu_S(\varphi_i) - S(\varphi_i)]^2 / \sum_{i=1}^{n} [\mu_S^*(\varphi_i) - S(\varphi_i)]^2 + w_2 \sum_{i=1}^{n} \sigma_S^2(\varphi_i) / \sum_{i=1}^{n} \sigma_S^{*2}(\varphi_i) \quad (21)$$

subject to

$$\mu_{\alpha_1} + k\sigma_{\alpha_1} \le 55^\circ \quad (22)$$

$$\mu_{\alpha_2} + k\sigma_{\alpha_2} \le 26^\circ, \quad (23)$$

where w_1 and w_2 are the weighting factors with $w_1 + w_2 = 1$. k is chosen to be 1 which indicates that with 84.13% probability, the constraint will be satisfied under the assumption that α_1 and α_2 are normally distributed. μ_S^* (obtained by $w_1 = 1$ and $w_2 = 0$) and σ_S^{*2} (obtained by $w_1 = 0$ and $w_2 = 1$) are the ideal solutions used to normalize the two aspects in robust design, i.e., optimizing the mean performance and minimizing performance deviations. Since the mechanism is desired to generate the output displacement following a specified function (Eqn. 16) over the whole range of $30^\circ \le \varphi \le 60^\circ$, multiple points (n=30), are used to evaluate the design performance. For the extreme condition approach, we have a similar optimization formulation.

$$Min\ F(x_1, x_2, x_3, x_4) = w_1 \sum_{i=1}^{n} [\overline{S}(\varphi_i) - S(\varphi_i)]^2 / \sum_{i=1}^{n} [\overline{S}^*(\varphi_i) - S(\varphi_i)]^2 + w_2 \sum_{i=1}^{n} \Delta S^2(\varphi_i) / \sum_{i=1}^{n} \Delta S^{*2}(\varphi_i) \quad (24)$$

subject to

$$\overline{\alpha}_1 + \Delta\alpha_1 \le 55^\circ \quad (25)$$

$$\overline{\alpha}_2 + \Delta\alpha_2 \le 26^\circ, \qquad (26)$$

where $\overline{S}^*$ (obtained by $w_1 = 1$ and $w_2 = 0$) and ΔS^{*2} (obtained by $w_1 = 0$ and $w_2 = 1$) are the ideal solutions used to normalize the two aspects in robust design.

To show the effect of the proposed method, the mechanism design solution using RSMs with the consideration of model uncertainty is compared with the one using the RSMs but without the consideration of uncertainties (called conventional optimization without uncertainty considerations). The comparison is made for using $w_1 = w_2 = 0.5$ as the weighting factors in the robust design formulations. It is noted from Table 1 that all the approaches generate feasible solutions where the maximum pressure angles are less than the required limits. The resulting displacement functions confirmed using the real analytical models at the design solution obtained from each of the three methods are shown in Fig. 9 for the range of input angles. It is noted that the functional curves generated by both the extreme condition approach and the statistical approach are closer to the desired location than the one based on the conventional optimization without uncertainty considerations. This indicates the benefits of using either the extreme condition approach or the statistical approach to the explicit modeling of the simulation errors.

Insert Table 1 here.

Insert Fig. 9 here.

The sum of the squares of the differences between the mean values of system outputs and the desired target values, i.e., $\sum_{i=1}^{30}[\mu_S(\varphi_i) - S(\varphi_i)]^2$ (called the mean square error), and the sum of the variances of system output $\sum_{i=1}^{30} \sigma_S^2(\varphi_i)$ are evaluated using Monte Carlo simulations at the design solutions obtained from the extreme condition approach, the statistical approach (with $w_1 = w_2 = 0.5$) and the conventional optimization without uncertainty considerations. The comparisons are provided in Figs. 10 and 11.

Insert Fig. 10 here.

Insert Fig. 11 here.

As shown in Figs. 10 and 11, conventional optimization using RSMs but without uncertainty considerations has the minimum mean square error, but the maximum variance. The maximum variance is caused by the model uncertainties $\varepsilon_\psi(\varphi, x_1, x_2)$, $\varepsilon_{\alpha_1}(x_1, x_2)$, $\varepsilon_S(\psi, x_3, x_4)$ and $\varepsilon_{\alpha_2}(x_3, x_4)$, which are ignored in the formulation of conventional optimization. This has resulted in the worst performance in meeting the displacement function requirement (see Fig. 9). When the variance is reduced with either the extreme condition approach or the statistical approach, the uncertainty is mitigated to a certain extent and more reliable design results can be obtained. As the case where $w_1 = w_2 = 0.5$, the variance is reduced from 2.0134 to 1.4320 with the extreme condition approach, and to 1.3956 with the statistic approach.

The trade-off between the mean square error and the variance in robust optimization can be treated by adjusting the weighting factors while maintaining $w_1 + w_2 = 1$. The weight setting of $w_1 = 1$ and $w_2 = 0$ yields a conventional optimization which does not consider any uncertainty and generates the lowest mean square error but maximum variance of system outputs. For this particular problem, with w_1 decreasing from 1 and w_2 increasing from 0, the variance of system outputs decreases and the mean square error increases. The achieved displacement function is shifted to the required function more closely. However, the decrease of variance slows down with a continued increase of w_2. As the mean square error increases accordingly, the proposed approaches with uncertainty considerations generate worse design results than the conventional optimization approach. This is especially true when w_1 is near 0 and w_2 is close to 1. As the value of design performance is influenced by both the mean location and its variance, how to deal with the trade-off between the mean square error and the variance in the robust design model is an important task and needs careful exercising.

V. Conclusions

An integrated methodology for propagating and mitigating the effect of uncertainties in simulation-based systems design is proposed in this paper. The *extreme condition approach* and the *statistical approach* are developed to propagate the effect of uncertainties and they are integrated with the proposed uncertainty mitigation strategy based on the principles of robust design. It is shown through the example that propagating and mitigating both external and internal uncertainties involved in simulation-based design will enable designers to make reliable decisions. The proposed methodology is flexible and comprehensive with ample potential for its application in the area of multidisciplinary collaborative systems design. Even though a simplistic simulation chain model is used for illustration and the details are based on a simple first order additive model for dealing with the internal uncertainty (see Eqns. 1 and 2), the concepts and principles presented in this paper can be extended to more complicated systems.

The computational efficiency of the extreme condition approach and the statistical approach will vary depending on the size of the problem (e.g., the number of design variables and the number of performance variables), the method for searching for extreme conditions, and data sample techniques when using the statistical approach. For complex engineering problems with "black box" type of simulation programs, it is generally recommend to use the statistical approach over the extreme condition approach. Also the statistical approach provides more information on the effect of uncertainty across the whole range of performance, while the later only deals with the conditions at extremes.

It should be noted that the effectiveness of the proposed method will depend on the quality of error models. If the simulation models deviate greatly from the real models or the error models do not describe the real situations very well, the proposed integrated method may generate unsatisfactory design results. For the example problem discussed in Section 4, the quality of the error model can be further improved in order to obtain better design solutions.

How to deal with the trade-off between the mean value and the variance in robust design is another important task that requires careful exercising. When choosing the weighting factors, the combination effect of the mean location and the variance needs to be considered with the preference of designers and their attitude towards risk.

The future work is targeted toward developing computationally efficient methods for propagating and mitigating the effect of uncertainty in a coupled multidisciplinary design environment where the performance prediction of one discipline may be the inputs of another discipline and vice versa. To reduce the computational effort in propagating the effect of uncertainties by Monte Carlo simulations, methodologies for fast and direct probabilistic evaluations of system performance need to be introduced and developed. In terms of decision making under uncertainty, more generic decision making model that is based on the concept of utility theory[33] will be developed to accommodate designer's preference and risk attitude.

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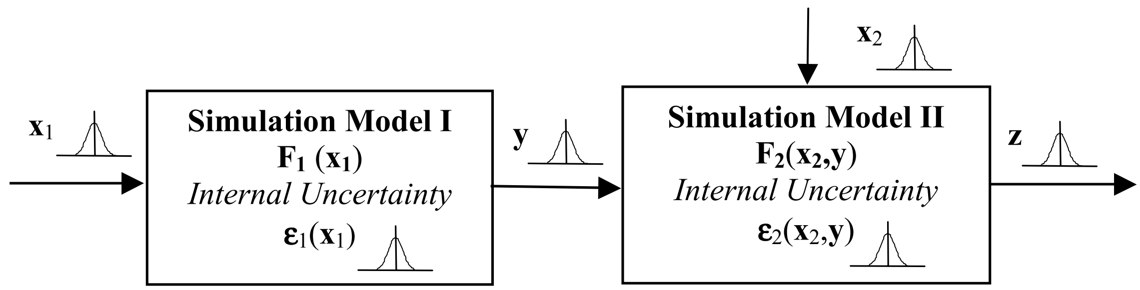

Figure 1 An illustrative simulation model chain

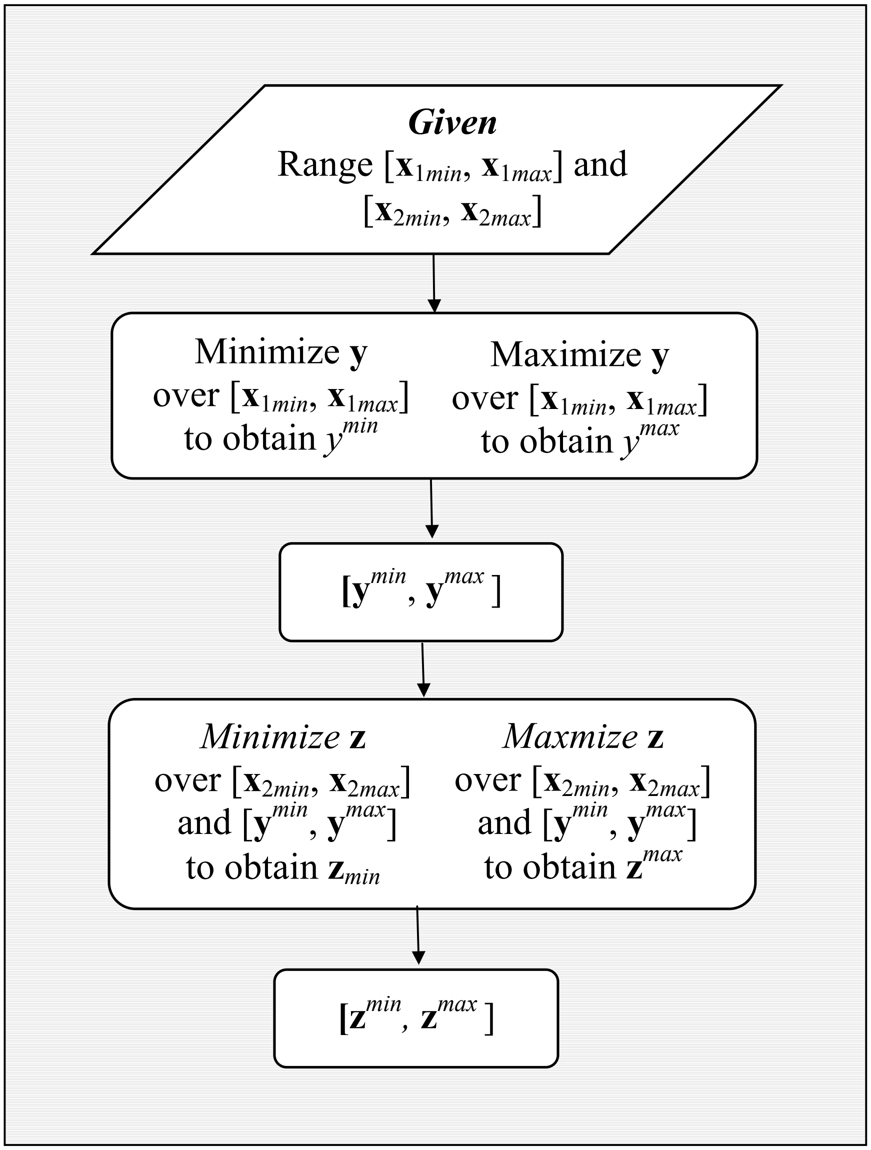

Figure 2 Procedure of the extreme condition approach

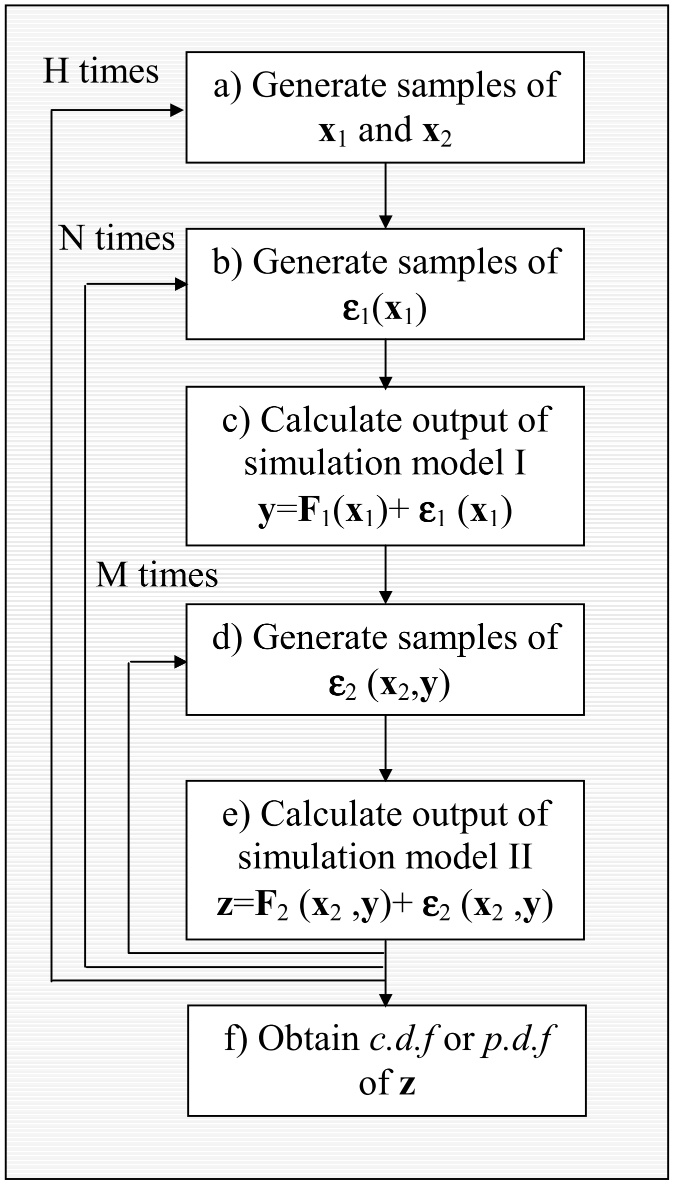

Figure 3 The process of Monte Carlo Simulation

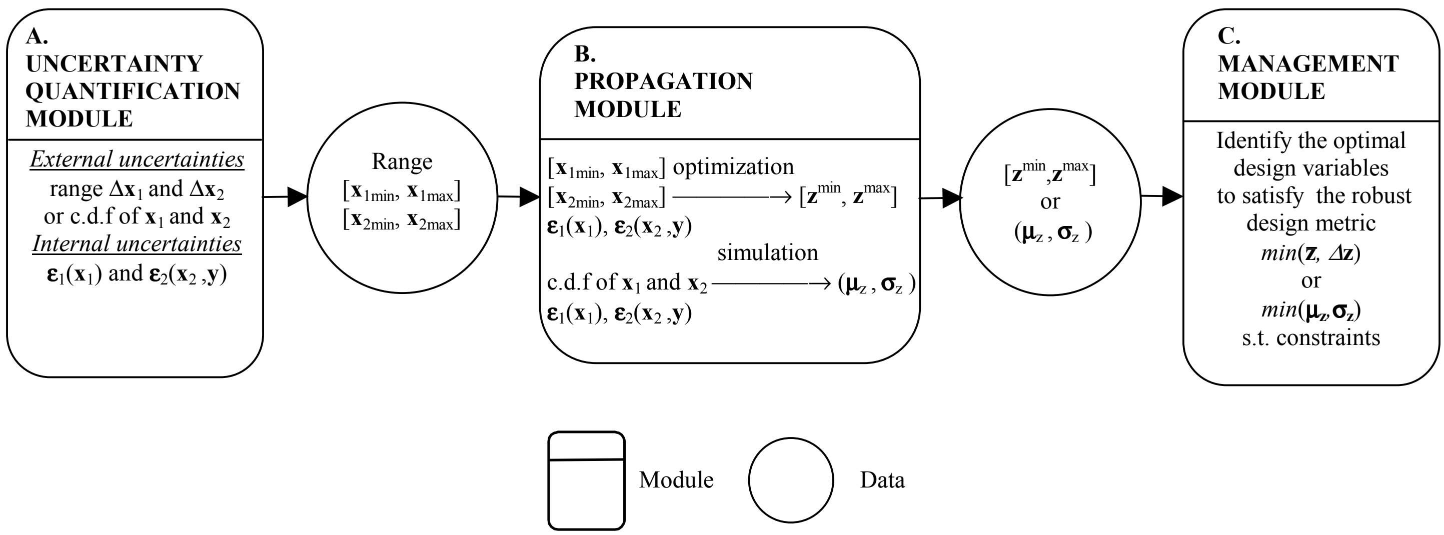

Figure 4 Integrated strategy for mitigating the effect of uncertainty

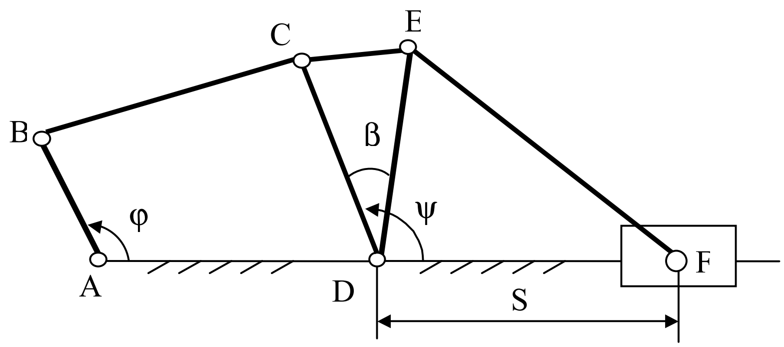

Figure 5. Six-link function-generator linkage

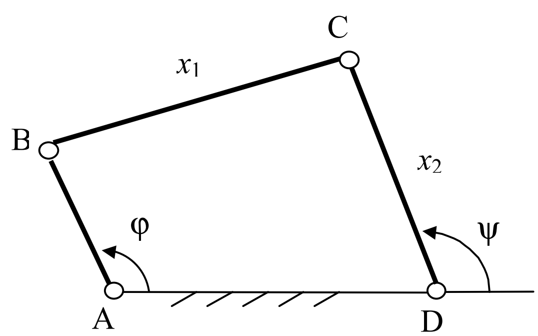

Fig. 6 The first four-bar linkage

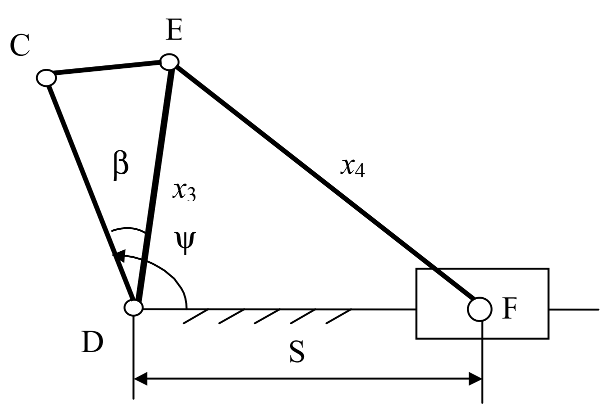

Fig. 7 The second four-bar linkage

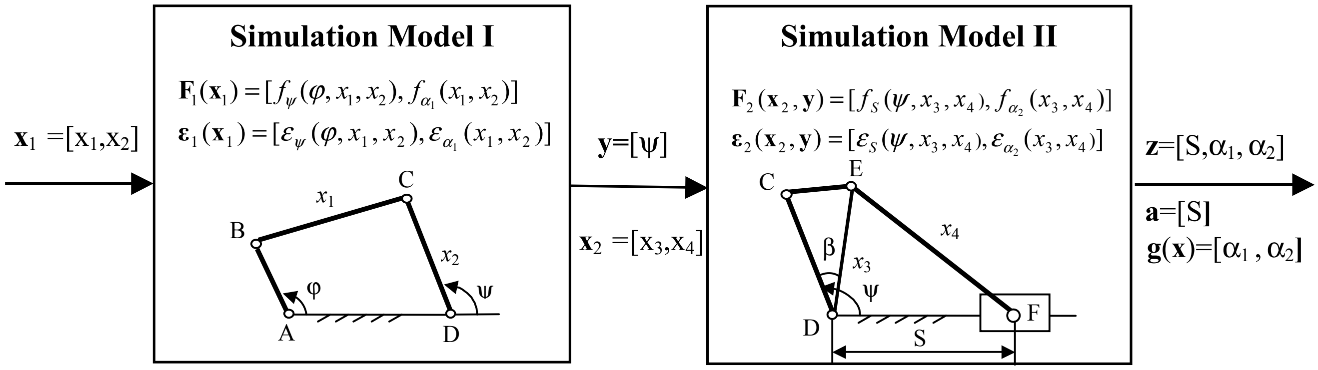

Figure 8 Simulation model for six-link function-generator linkage

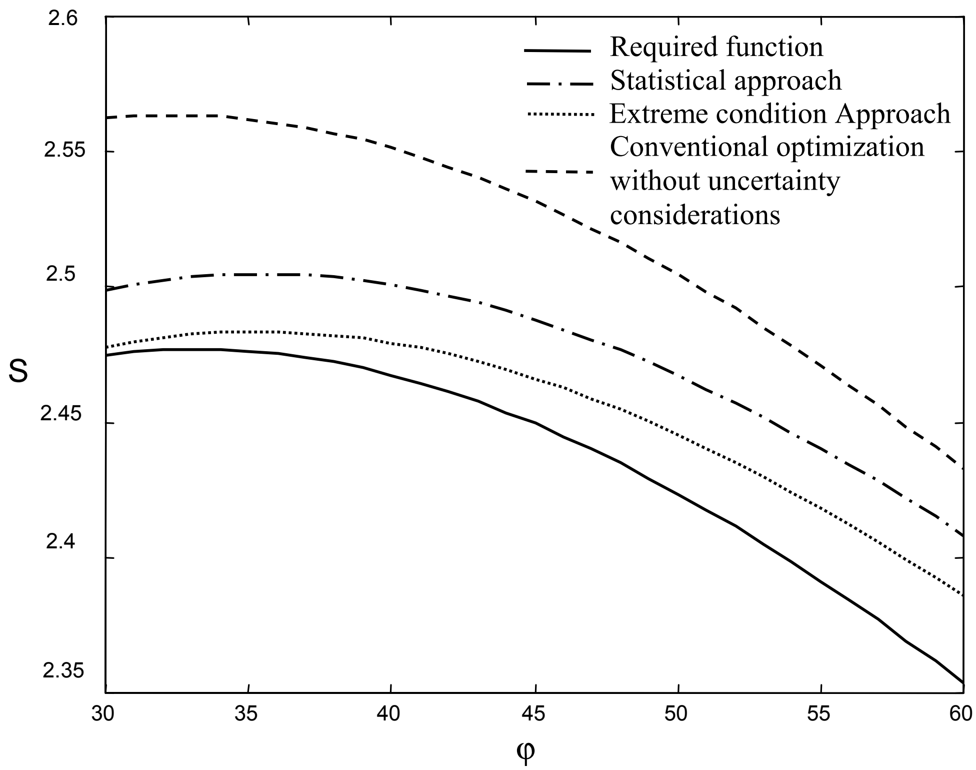

Figure 9 Comparison of design results

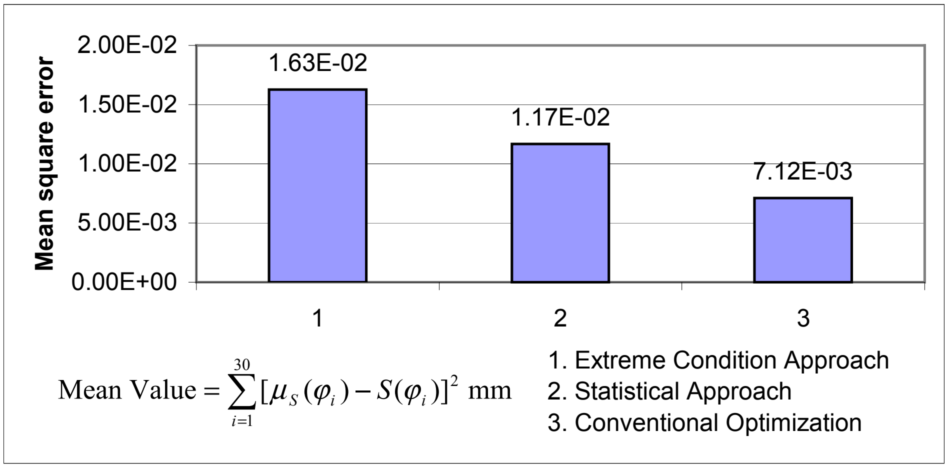

Figure 10 Confirmed mean square error

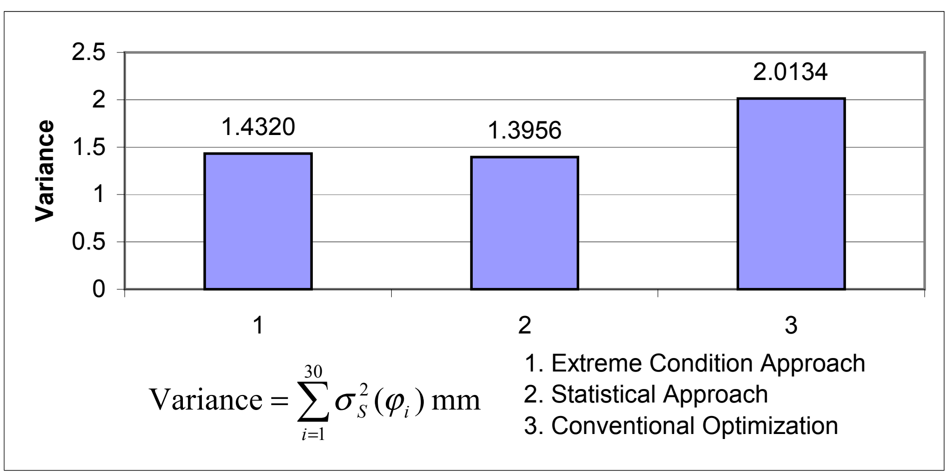

Figure 11 Confirmed variance

Table 1 Comparisons of Design Results

Method	x_1 (mm)	x_2 (mm)	x_3 (mm)	x_4 (mm)	α_1 (°)	α_2 (°)
Extreme Condition Approach	1.774	1.537	1.096	2.553	53.921	25.431
Statistical Approach	1.775	1.542	1.096	2.575	53.950	25.199
Conventional Optimization (without uncertainty considerations)	1.831	1.478	1.098	2.527	55.013	25.769